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Abstracts of Reports

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Abstracts of Reports

Topological properties of the open extension topology

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Open extension topology is introduced in [1] for the case when its carrier differs from the one of the starting topological space by one point. A particular case of this construction is excluded point topology which appears as an open extension of the discrete topology. The most famous example of excluded point topology is the Sierpinski space. We study topological properties of the generalization of this construction to the case of an arbitrary superset of the carrier of the original space.

Let (X, τ) be a topological space and let X^* be a superset of X. Then the family $\tau^* = \{U \subset X^* \mid U \supset X \text{ or } U \in \tau\}$ is a topology for X^*, which is called *the open extension topology of X to X^**.

The space X^* is first countable (separable) if and only if X is first countable (separable). The space X^* is second countable if and only if X is second countable and the complement $X^* \backslash X$ is at most countable. The space X^* is compact (Lindelöf) if and only if the complement $X^* \backslash X$ is finite (at most countable).

Let $X^* \neq X$. Then the space X^* is path connected. The space X^* is T_0 if and only if X is T_0. The space X^* is T_4 if and only if $|X^* \setminus X| = 1$. The space X^* is not T_i for $i = 1, 2, 3$. In particular, X^* is neither regular, normal, nor metrizable.

References

1. L. A. Steen and J. A. Seebach, Jr., *Counterexamples in topology*, Dover Publications, New York, 1978.

2. V. M. Babych and V. O. Pyekhtyeryev, *Open extension topology*, Proc. Intern. Geom. Center. **8**:2 (2015) 20–25 (in Ukrainian).

Topological spaces with an ω^ω-base

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Given a partially ordered set P, we say that a topological space X has a *local P-base* if each point $x \in X$ has a neighborhood base $(U_\alpha[x])_{\alpha\in P}$ such that $U_\beta[x] \subset U_\alpha[x]$ for all $\alpha \le \beta$ in P. For every $\alpha \in P$ the neighborhoods $U_\alpha[x]$, $x \in X$, compose an entourage $U_\alpha = \{(x,y) \in X \times X : y \in U_\alpha[x]\}$ on X. The indexed family $\{U_\alpha\}_{\alpha\in P}$ is called a *P-base* for the topological space X. A P-base $\{U_\alpha\}_{\alpha\in P}$ is called *locally uniform* if for any point $x \in X$ and neighborhood $O_x \subset X$ of x there is $\alpha \in P$ such that the ball $U_\alpha U_\alpha^{-1} U_\alpha[x] = \{y \in X : (x,y) \in U_\alpha U_\alpha^{-1} U_\alpha\}$ is contained in O_x.

It is clear that a topological space is first-countable if and only if it has an ω-base. By Moore Metrization Theorem, a topological space X is metrizable if and only if X is a T_0-space with a locally uniform ω-base.

In the talk we shall discuss some properties of topological spaces possessing a (locally uniform) ω^ω-base.

We show that topological spaces with an ω^ω-base share some common properties with first-countable spaces. In particular, many known upper bound on the cardinality of first-countable spaces remain true for countably tight spaces with an ω^ω-base.

On the other hand, topological spaces with a locally uniform ω^ω-base have many properties, typical for generalized metric spaces.

Paper reference: T. Banakh, *Topological spaces with an ω^ω-base*, preprint (**http://arxiv.org/abs/1607.07978**).

Isometric copies of directed trees in orientations of graphs

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For every $n \in \mathbb{N}$ we construct a finite graph G whose every orientation contains an isometric copy of any oriented tree on n vertices, and evaluate the smallest possible cardinality of G (as $o(n^{4n})$).

On the other hand, we prove that every graph G admits an orientation containing no directed ω-paths of infinite diameter.

Paper reference: **http://arxiv.org/abs/1606.01973**

On the embeddings and closures of the topological λ-polycyclic monoid

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We prove that for every cardinal $\lambda \geqslant 2$ any continuous homomorphism from a topological semigroup P_λ into an arbitrary countably compact topological semigroup is annihilating and there exists no a Hausdorff feebly compact topological semigroup which contains P_λ as a dense subsemigroup. We give sufficient conditions when a topological inverse λ-polycyclic monoid P_λ is absolutely H-closed in the class of topological inverse semigroups and construct an example of a topological inverse monoid S which contains the polycyclic monoid P_2 as a dense discrete subsemigroup.

On generating the classical ternary Cantorval

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We show that the Cantorval connected with the ternary Cantor set is not an achievement set (i.e. the set of subsums) of any series. However, it is an attractor of IFS consisting of 17 affine functions.

Geodesic mappings onto symmetric manifolds

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Geodesic mappings of manifolds with affine connection onto symmetric manifolds were studied. Major equations of the aforementioned mappings were obtained in close differential equations system of Cauchy type in covariant derivatives. The number of essential parameters the general solution depends on was defined.

Lattices of topologies

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The lattice $L(X)$ of all topologies on a given set X was introduced by Garrett Birkhoff in 1936. For two topologies in $L(X)$ the join is generated by the union whereas the meet is just the intersection of those topologies. Elements of $L(X)$ are complementary whenever the join is the greatest and the meet is the smallest topology in $L(X)$. We are interested in the sublattice $L_1(X)$ consisting of all T_1-topologies and looking for complementary topologies with properties like compactness or at least the Hausdorff separation property.

On spaces of weakly discontinuous functions

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A function $f : X \to Y$ between topological spaces is *weakly discontinuous* if for every non-empty subspace A of X the set $D(f|A)$ of discontinuity points of the restriction $f|A$ is nowhere dense in A. By $WD_p(X)$ we denote the space of all weakly discontinuous real-valued functions on X endowed with the topology of pointwise convergence.

In this report we focus on the results related to the function spaces $WD_p(X)$ and their subspaces.

A new variant of the game of cops and rober

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Preservation of weakly discontinuous functions by topological functors

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Let $F\colon \mathsf{Comp} \to \mathsf{Comp}$ be a monomorphic functor in the category of compact Hausdorff spaces and their continuous maps. We say that F has *finite supports* (resp. *finite degree* $\le n$) if for every compact space X and $a \in FX$ there exists a continuous map $f\colon D \to X$ defined on a finite discrete space D (of cardinality $|D| \le n$) such that $a \in Ff(FD)$.

We prove that each monomorphic functor $F\colon \mathsf{Comp} \to \mathsf{Comp}$ with finite supports admits an extension $\bar{F}\colon \mathsf{Tych} \to \mathsf{Tych}$ to the category Tych whose objects are Tychonoff spaces and morphisms are arbitrary (not necessarily continuous) functions between Tychonoff spaces.

If the functor F has finite degree, then the extended functor $\bar{F}$ preserves weakly discontinuous maps (a function $f\colon X \to Y$ between topological spaces is called *weakly discontinuous* if each subspace $A \subset X$ contains a dense open subset $U \subset A$ such that the restriction $f|U$ is continuous).

The epi-reflective hull of the class all metric uniform spaces of a given weight

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This talk is a continuation of authors talk "Ultrafilter-completeness on a zero-sets of uniformly continuous functions" on Toposym 2016 (Prague, Czech Republic, July 25–29, 2016), which is available on the website of Toposym 2016 and Book of Abstracts, p.76.

For an infinite cardinal τ by $\mathcal{M}_\tau$ we denote the class of all metric uniform spaces of topological weight $\leq \tau$ and by $\mathfrak{L}(\mathcal{M}_\tau)$ the epi-reflective hull of the class $\mathcal{M}_\tau$ in a category $ZUnif$ whose an objects are all uniform spaces, and a morphisms are all coz-mappings. A mapping $f : uX \to vY$ between uniform spaces uX and vY is said to be a *coz-mapping*, if $f^{-1}(\mathcal{Z}_v) \subseteq \mathcal{Z}_u$, where $\mathcal{Z}_u(\mathcal{Z}_v)$ are zero-sets of all uniformly continuous functions on $uX(vY)$ or $f^{-1}(C\mathcal{Z}_v) \subseteq C\mathcal{Z}_u$, where $C\mathcal{Z}_u = \{X \backslash Z : Z \in \mathcal{Z}_u\}$ ($C\mathcal{Z}_v = \{Y \backslash Z : Z \in \mathcal{Z}_v\}$) is the family of complements [Z. Frolik, M. Charalambous]. If $Y = \mathbb{R}$ or $Y = I$, then a coz-mapping $f : uX \to \mathbb{R}$ is called a *u-continuous function* and a coz-mapping $f : uX \to I$ is said to be a *u-function.* Following M. Charalambous, a set in $\mathcal{Z}_u$ ($C\mathcal{Z}_u$) is called *u-closed* (*u-open*).

We denote by $C_u(X)$ (resp. $C^*_u(X)$) the set of all (bounded) u-continuous functions on a uniform space uX. It is known that $C_u(X)$ forms an *algebra with inversion* [Chekeev] in sense Hager-Johnson, Isbell.

Let X be a subspace of a Tychonoff space Y and u be a uniformity on X, v be a uniformity on Y such that $\mathcal{Z}_v \wedge X = \mathcal{Z}_u$. The uniform space uX is said to be *C_u-embedded* (res. *C^*_u-embedded*) in the uniform space vY, if any function of $C_u(X)$ ($C^*_u(X)$) can be extended to a function in $C_v(Y)$ ($C^*_v(Y)$).

A maximal centered system of $\mathcal{Z}_u$-zero-sets is said to be *z_u-ultrafilter.* For a z_u-ultrafilter p the family $co(p) = \{X \setminus Z : Z \in p\}$ is called *τ-locally finitely additive*, if $\cup\alpha \in co(p)$ for every locally finite subfamily $\alpha \subset co(p)$ of cardinality

$|\alpha| \leq \tau$. Every z_u-ultrafilter p with τ-locally finitely additive family $co(p)$ is called a *τ-weakly Cauchy z_u-ultrafilter.*

The name for a τ-weakly Cauchy z_u-ultrafilter is due to the fact that every Cauchy z_u-ultrafilter with respect uniformity whose base consists of all locally finite *coz*-additive u-open coverings of cardinality $\leq \tau$ has the τ-locally finite additive property and it is countably centered and vise versa.

A uniform space uX is called *τ-weakly z_u-complete*, if every τ-weakly Cauchy z_u-ultrafilter converges.

Theorem 1. *A uniform space uX is τ-weakly z_u-complete if and only if $uX \in \mathfrak{L}(\mathcal{M}_\tau)$.*

The family $\mathcal{Z}_u$ is a normal base in sense O.Frink, and the Wallman compactification $\omega(X, \mathcal{Z}_u)$ is a β-like compactification in sense Mrówka. Moreover, it has the next property, similar to Stone–Čech compactification [A. Chekeev].

Theorem 2. *For every uniform space uX the Wallman compactification $\omega(X, \mathcal{Z}_u) = \beta_u X$ is β-like compactification with the next equivalent properties:*

- (I) *Every coz-mapping f from uX into any compactum K has a continuous extension $\beta_u f$ from $\beta_u X$ into K.*
- (II) *uX is C^*_u-embedded in $\beta_u X$.*
- (III) *Any two disjoint u-closed sets in uX have disjoint closures in $\beta_u X$.*
- (IV) *For any two u-closed sets Z_1 and Z_2 in uX the equality $[Z_1 \cap Z_2]_{\beta_u X} = [Z_1]_{\beta_u X} \cap [Z_2]_{\beta_u X}$ holds.*
- (V) *Distinct z_u-ultrafilters on uX have distinct limits in $\beta_u X$.*

The compactification $\beta_u X$ is unique in the next sense: if a compactification Y of uX satisfies anyone of listed conditions, then there exists a homeomorphism of $\beta_u X$ onto Y that leaves X pointwise fixed.

The *Wallman τ-completion* $\mu_u^\tau X$ of a uniform space uX is the subspace of $\beta_u X$ consisting of the set of all τ-weakly Cauchy z_u-ultrafilteres on $\mathcal{Z}_u$.

Theorem 3. *Every uniform space uX has Wallman τ-completion $\mu_u^\tau X$, contained in a β-like compactification $\beta_u X$ with the next equivalent properties:*

- (I) *Every coz-mapping f from uX into any τ-weakly z_u-complete uniform space vY has a coz-mapping extension $\tilde{f}$ from $\mu_u^\tau X$ into vY.*
- (II) *Every coz-mapping f from uX into an arbitrary metric uniform space $u_\rho M$ of weight $\leq \tau$ has a coz-mapping extension $\tilde{f}$ from $\mu_u^\tau X$ into $u_\rho M$.*
- (III) *If $\{Z_i\}_{i\in I}$ is a family of u-closed sets of cardinality $\leq \tau$ with $\{X \setminus Z_i\}_{i\in I}$ locally finite, and $\bigcap_{i\in I} Z_i = \emptyset$, then $\bigcap_{i\in I} [Z_i]_{\mu_u^\tau X} = \emptyset$.*

(IV) *If* $\{Z_i\}_{i\in I}$ *is a family of u-closed sets of cardinality* $\leq \tau$ *with* $\{X \setminus Z_i\}_{i\in I}$ *locally finite, then* $\bigcap_{i\in I}[Z_i]_{\mu_u^\tau X} = \left[\bigcap_{i\in I} Z_i\right]_{\mu_u^\tau X}$.

(V) *Every point of* $\mu_u^\tau X$ *is the limit of unique* τ*-weakly Cauchy* z_u*-ultrafilter.*

(VI) $\mu_u^\tau X$ *is a completion of* X *with respect to a uniformity whose base consists of all locally finite coz-additive u-open coverings of cardinality* $\leq \tau$.

The Wallman τ*-completion* $\mu_u^\tau X$ *is unique in the next sense: If a uniform space* vY *is an extension of* uX *satisfies anyone of listed conditions, then there exists a coz-homeomorphism of* $\mu_u^\tau X$ *onto* vY *that leaves* X *pointwise fixed.*

By Z. Frolik, a bijective mapping $f : uX \to vY$ between uniform spaces is called a *coz-homeomorphism*, if both f and f^{-1} are *coz*-mappings. A uniform spaces uX and vY are *coz-homeomorphic*, if there exists a *coz*-homeomorphism of uX onto vY.

Theorem 4. *For a uniform space* uX *the next conditions are equivalent:*

(1) uX *is* τ*-weakly* z_u*-complete;*

(2) X *is complete with respect to the uniformity whose base consists of all locally finite coz-additive u-open coverings of cardinality* $\leq \tau$*;*

(3) $uX = \mu_u^\tau X$*;*

(4) uX *is coz-homeomorphic to a closed uniform subspace of a product of metric uniform spaces of the class* $\mathcal{M}_\tau$.

Thus, every τ-weakly z_u-complete uniform space is a closed uniform subspace of product from the class $\mathcal{M}_\tau$, hence the class of all τ-weakly z_u-complete uniform spaces in the category *ZUnif* coincides with epi-reflective hull $\mathfrak{L}(\mathcal{M}_\tau)$ [Franklin, Herrlich, Hager, Vilímovský]. For any uniform space uX the Wallman τ-completion $\mu_u^\tau X$ is a projective object in $\mathfrak{L}(\mathcal{M}_\tau)$, i.e. $\mu_u^\tau X$ is the essentially unique τ-weakly z_u-complete uniform space containing X densely such that each *coz*-mapping $f : uX \to vY$ $(vY \in \mathfrak{L}(\mathcal{M}_\tau))$ admits a *coz*-mapping extension $\mu_u^\tau f : \mu_u^\tau X \to vY$, or $\mu_u^\tau : uX \to \mu_u^\tau X$ is an epi-reflection and *coz*-homeomorphic embedding.

Ramsey multiplicity and Ramsey trees

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Let $D_k(G)$ be the number of complete subgraphs on k vertices of a graph G on n vertices. The object of study is the Ramsey multiplicity constants c_k

$$c_k = \lim_{n\to\infty} \binom{n}{k}^{-1} \cdot \min_{G,\ |V(G)|=n} D_k(G) + D_k(\bar{G}).$$

Consider the set $T_{(m,n)}$ of binary words in a 2-letter alphabet $\{left, right\} = \{l, r\}$ with less than m letters l and less than n letters r. For a binary word w consider the words $l(w) = wl$,$r(w) = wr$ and a parent word $p(w)$ such that $w = l(p(w))$ or $r(p(w))$. A word w will be called *left* (*right*) if w equals $l(p(w))$ (resp. $r(p(w))$. Let i be the function assigning to each word its last letter.

By an (m, n)-*Ramsey tree* we mean a labeling of a set $T_{(m,n)}$, i.e. a map $L_T \colon T_{(m,n)} \to [0, 1]$ such that $L(w) = L(l(w)) + L(r(w))$ and for the empty word $L(\varnothing) = 1$. For the maximal word $w \in T$ define weight of w as $M(w) = \prod\limits_{i(v)=i(w)} L(p(v))L(w)$.

For a (m, n)-Ramsey tree T define the multiplicity $M_{tree}(C_l, C_r, m, n, T)$ as

$$M_{tree}(C_l, C_r, m, n, T) = \min_{w\in\max T} C_{i(w)} M(w).$$

It will be proven that the Ramsey multiplicity constants are greater than the respective multiplicities of Ramsey trees.

Covering properties of filters and the Mathias-Prikry forcing

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Some properties of the Mathias-Prikry forcing $\mathbb{M}(F)$ (a filter F is a parameter) can be characterized by topological covering properties of the filter F as a subspace of the Cantor space. E.g., $\mathbb{M}(F)$ is weakly bounding iff F is Menger, and $\mathbb{M}(F)$ is almost bounding if F is Hurewicz. The talk will explore these connections.

The presented results are joint work with L. Zdomskyy and D. Repovš, and O. Guzman and M. Hrušak.

The Lwów period in Kuratowski's life

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A survey of biographical information and mathematical results of Kazimierz Kuratowski in the period of his professorship at Lwów Polytechnic (1927–1933).

Semigroups of k-linked upfamilies

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The through study of various extensions of semigroups was started in [9] and continued in [1–8, 10–12]. The largest among these extensions is the semigroup $\upsilon(S)$ of all upfamilies on a semigroup S. A family $\mathcal{M}$ of non-empty subsets of a set X is called an *upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. Each family $\mathcal{B}$ of non-empty subsets of X generates the upfamily

$$\langle B \subset X : B \in \mathcal{B} \rangle = \{A \subset X : \exists B \in \mathcal{B} (B \subset A)\}.$$

A family $\mathcal{F}$ of non-empty subsets of a set X that is closed under taking supersets and finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Cech compactification* of X, see [13]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Identifying each point $x \in X$ with the principal ultrafilter $\langle\{x\}\rangle$ we obtain the inclusions $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [9] that any associative binary operation $*\colon S \times S \to S$

can be extended to an associative binary operation $\circ\colon \upsilon(S)\times\upsilon(S)\to\upsilon(S)$ by the formula

$$\mathcal{L}\circ\mathcal{M}=\Big\langle \bigcup_{a\in L} a*M_a : L\in\mathcal{L},\ \ \{M_a\}_{a\in L}\subset\mathcal{M}\Big\rangle$$

for upfamilies $\mathcal{L},\mathcal{M}\in\upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains many other important extensions of S. In particular, it contains the semigroups $N_k(S)$ of k-linked upfamilies for $k\in\mathbb{N}\setminus\{1\}$. An upfamily $\mathcal{L}\in\upsilon(S)$ is called *k-linked* if $\bigcap\mathcal{F}\neq\emptyset$ for any subfamily $\mathcal{F}\subset\mathcal{L}$ with $|\mathcal{F}|\le k$.

Given a group G we we shall discuss the algebraic structure of the extension $N_k(G)$ of G. We describe right and left zeros, idempotents, the minimal ideal, left cancelable and right cancelable elements of the semigroup $N_k(G)$ of k-linked upfamilies and characterize groups G whose extensions $N_k(G)$ are commutative.

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Dense free subgroups of automorphism groups of homogeneous partially ordered sets

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Let $1 \leq n \leq \omega$. Let A_n be the set of natural numbers less than n. Define $<$ on A_n so that for no $x, y \in A_n$ is $x < y$. Let $B_n = A_n \times \mathbb{Q}$ where $\mathbb{Q}$ is the set of rational numbers. Define $<$ on B_n so that $(k, p) < (m, q)$ iff $k = m$ and $p < q$. Let $C_n = B_n$ and define $<$ on C_n so that $(k, p) < (m, q)$ iff $p < q$. Finally, let $(D, <)$ be the universal countable homogeneous partially ordered set, that is a Fraïssé limit of all finite partial orders.

A structure is called *ultrahomogeneous*, if every embedding of its finitely generated substructure can be extended to an automorphism. Schmerl in [1] showed that there are only countably many, up to isomorphism, ultrahomogeneous countable partially ordered sets. More precisely he proved the following characterization.

Theorem. *Let $(H, <)$ be a countable partially ordered set. Then $(H, <)$ is ultrahomogeneous iff it is isomorphic to one of the following:*

- (a) $(A_n, <)$ *for* $1 \leq n \leq \omega$;
- (b) $(B_n, <)$ *for* $1 \leq n \leq \omega$;
- (c) $(C_n, <)$ *for* $2 \leq n \leq \omega$;
- (d) $(D, <)$.

Moreover, no two of the partially ordered sets listed above are isomorphic.

Consider automorphisms groups $\mathrm{Aut}(A_\omega) = S_\infty$, $\mathrm{Aut}(B_n)$, $\mathrm{Aut}(C_n)$ and $\mathrm{Aut}(D)$. We prove that each of these groups contains two elements f, g such that the subgroup generated by f and g is free and dense. By Schmerl's Theorem the automorphism group of a countable infinite partially ordered set is freely topologically 2-generated.

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Changeable sets and their possible applications

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We study a new class of abstract mathematical objects: changeable sets [1–3]. From a formal point of view, changeable sets are sets of objects which, unlike the elements of ordinary (static) sets, may be in the process of continuous transformations, and which may change properties depending on the point of view on them (that is on reference frame). Changeable sets can be interpreted as a mathematical abstraction of the evolution models for physical, biological, and other systems in macrocosm. That is why, the theory of changeable sets is closely connected with the famous sixth Hilbert problem. It should be noted, that for the construction of the theory of changeable sets it is not necessary to review or complement axiomatic foundations of classical set theory. Changeable sets are defined as a new abstract universal class of objects within the framework of classical set theory (just as rings, fields, lattices, linear spaces, etc.). In the paper [3] the universal kinematics were constructed on the basis of the theory of changeable sets. Universal kinematics are the mathematical objects, which consist of changeable sets and their geometrical environment, represented by different metric, topological, linear, Banach, Hilbert and other spaces together with the universal coordinate transforms between reference frames. This abstract approach may be interesting for astrophysics, because there exists a hypothesis, that in large scale of the Universe, physical laws (in particular, the laws of kinematics) may be different from the laws, acting in the neighborhood of our solar System. For example we have proved some abstract theorems, which establish the link between time-irreversibility of some universal kinematics (i.e., absence temporal paradoxes in it) and the structure of its coordinate transforms [4].

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Graph coloring problems with number theoretic flavor

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I will present some graph coloring problems with intensive number theoretic flavor. One of my favorite goes as follows. Suppose that each vertex v of a graph G is assigned with a positive integer n_v. For an unknown reason, we multiply n_v by the degree d_v of a vertex v, thereby obtaining a new number $m_v = n_v \cdot d_v$. If we did so for all vertices, then it may happen that new numbers constitute a proper coloring of G, that is, $m_v \neq m_u$ whenever v and u are adjacent. The initial coloring with numbers n_v is then called *ironic*. Now, what is the least possible upper bound $\varphi(G)$ for the maximum color in an ironic coloring of G? We conjecture that $\varphi(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number of a graph G. I will demonstrate that the conjecture is asymptotically true, and also point on its unexpected (or expected?) connections to some famous number theoretic problems.

Sala Weinlös and her doctoral thesis in Lwów University

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We shall discuss the biography and shientific work of Sala Weinlös (1906-194?), a less known student of Hugo Steinhaus. In 1927 she defended her Doctoral Thesis "O niezaleznosci I, II, i IV grupy aksjomatów geometrii euklidesowej trójwymiarowej" ("On the independence of I, II, and IVth groups of axioms of the three-dimensional Euclidean Geometry"). She published two papers.

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On locally compact semitopological 0-bisimple inverse ω-semigroups

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We shall follow the terminology of [1,2,4]. An inverse semigroup S is called a 0-*bisimple ω-semigroup* if S has two $\mathscr{D}$-classes: $S \setminus \{0\}$ and $\{0\}$, and the subset of idempotents of $S \setminus \{0\}$ is order isomorphic to the ω-chain. The results of Lallement and Petrich [3] imply that every 0-bisimple inverse ω-semigroup is isomorphic to the Reilly semigroup $\mathbf{B}(G,\theta)^0$ with adjoined zero.

We describe the structure of Hausdorff locally compact semitopological 0-bisimple inverse ω-semigroups with compact groups of units. In particular, we show that a Hausdorff locally compact semitopological 0-bisimple inverse ω-semigroup with a compact maximal subgroup is either compact or topologically isomorphic to the topological sum of its $\mathscr{H}$-classes. Also we describe the structure of Hausdorff locally compact semitopological 0-bisimple inverse ω-semigroups whose group of units is isomorphic to the additive group of integers.

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Algorithmic geometric problems connected with a manipulator motion

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Motivated by the problems of modelling a manipulator motion in computer graphics, we shall discuss the following algorithmic problems.

Problem 1. Given a sequence of complex numbers $(z_k)_{k=1}^n$ with $\sum_{k=1}^n z_k = a$ and a complex number $\tilde{a}$ find a sequence of complex numbers $(\tilde{z}_k)_{k=1}^n$ such that $\sum_{k=1}^n \tilde{z}_k = \tilde{a}$ and $|\tilde{z}_k| = |z_k|$ for all k.

Problem 2. Given a sequence of complex numbers $(z_k)_{k=1}^n$ and a sequence of closed intervals $(I_k)_{k=1}^n, I_k = [l_k, u_k]$ with $\sum_{k=1}^n z_k = a$, $arg(z_{k+1}z_k^{-1}) \in I_k$ for all k and a complex number $\tilde{a}$ find a sequence of complex numbers $(\tilde{z}_k)_{k=1}^n$ such that $\sum_{k=1}^n \tilde{z}_k = \tilde{a}$, $|\tilde{z}_k| = |z_k|$ and $arg(\tilde{z}_{k+1}\tilde{z}_k^{-1}) \in I_k$ for all k.

Problem 3. If it is impossible to find such a sequence $(\tilde{z}_k)_{k=1}^n$ for second problem, then find the closest point $\tilde{a}'$ to the point $\tilde{a}$ that has solution in terms of second problem. It is also desirable that the points $\tilde{z}_k$ depend continuously on $\tilde{a}$.

Continuation of functions, defined on the circle, to optimal Morse functions on the surface

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Let M be an oriented surface of genus g with one component of the boundary ∂M and $f : M \to \mathbb{R}$ be a Morse function with n critical points belonging to the boundary and critical values $1, 2, \dots, n$. We assume that f has at most one critical point on each level-line. To such function f we shall assign a substitution $(a_1, a_2, \dots, a_n)$.

Morse function, defined on a surface with boundary is called an *mm-function* if its restriction to the boundary is a Morse function and all critical points belong to the boundary of the manifold. An mm-function is *optimal* if it has the least possible number of critical points among all mm-functions on a given surface.

Let f be a Morse function defined on an oriented surface of genus g with one component of the boundary and $(a_1, a_2, \dots, a_n)$ be the corresponding substitution. If the restriction of f to the boundary can be extended to the whole surface to an optimal mm-function with one-connected components of level-lines, then the substitution $(a_1, a_2, \dots, a_n)$ satisfies the following conditions:

- (i) numbers $1, 2, 3$ are minimum numbers;
- (ii) among numbers $4, 5, \dots, 4k + 3$ there exist $2k$ minimum numbers for all possible k.

Paper reference: B. I. Hladysh and O. O. Prishlyak, *Functions with nondegenerate critical points on the boundary of the surface*, Ukr. Mat. Zh. **68**:1 (2016), 28–37 (in Ukrainian).

Almost optimal aproximations of capacities on metric compacta whith capacities from a closed I-convex subspace

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A triple $(X, \vee, \wedge)$ is called an *I-convex compactum* [2] if X is a compact Hausdorff space with a Lawson continuous pairwise I-convex combination $X \times I \times X \to X$, $(x, \alpha, y) \mapsto x \vee \alpha \wedge y$, which (for $\alpha = 1$) makes X a compact Hausdorff Lawson upper semilattice [3].

For a compact Hausdorff space X by $\exp X$ we denote the hyperspace of all closed subsets of X.

A function $c : \exp X \cup \{\emptyset\} \to I$ is called [5] a *capacity* on X if has the following three properties:

(1) $c(\emptyset) = 0$;
(2) if $c(F) \le c(G)$ for any closed subset $F \subset G$ in X (the *monotonicity*);
(3) if a closed subset $F \subset X$ has $c(F) < a$, then F has an open neighborhood O_F in X such that $c(K) < a$ for any compact subset $K \subset O_F$ (the *upper semicontinuity*).

If, additionally, $c(X) = 1$ (or $c(X) \le 1$), then the capacity is called *normalized* (resp. *subnormalized*).

By MX and $\underline{M}X$ denote the sets of all normalized and of all subnormalized capacities respectively. The space $\underline{M}X$ carries a compact Hausdorff topology [5] generated by the subbase consisting of the sets

$$O_-(F, a) = \{c \in \underline{M}X \mid c(F) < a\}$$

and

$$\begin{aligned} O_+(U, a) &= \{c \in \underline{M}X \mid c(U) > a\} = \\ &= \{c \in \underline{M}X \mid c(K) > a \text{ for some compact subset } K \subset U\}, \end{aligned}$$

where $a \in I$ and F, U are closed and open sets in X, respectively.

The space $\underline{M}X$ endowed with the operations

$$\vee : \underline{M}X \times \underline{M}X \to \underline{M}X \colon (c_1, c_2) \mapsto \max\{c_1, c_2\}$$

and $\wedge : I \times \underline{M}X \to \underline{M}X \colon (\alpha, c) \mapsto \min\{\alpha, c\}$, is an I-convex compactum.

If the topology of the compact space X is generated by a metric d, then the topology of the space $\underline{M}X$ is generated by the metric

$$\hat{d}(c, c') = \inf\big\{\varepsilon > 0 \mid \forall F \in \exp X \;\; c(\bar{O}_\varepsilon(F)) + \varepsilon \geq c'(F), c'(\bar{O}_\varepsilon(F)) + \varepsilon \geq c(F)\big\},$$

where $\bar{O}_\varepsilon(F)$ is the closed ε-neighborhood of a subset $F \subset X$. Hence $(\underline{M}X, \hat{d})$ is a metric compactum and $(\underline{M}X, \vee, \wedge)$ is a *metric I-convex compactum.*

We approximate a capacity $c \in \underline{M}X$ with capacities that belong to a closed I-convex subspace $S \subset \underline{M}X$. The I-convexity means that S contains all I-convex combinations of the form $\bigvee_{i \in A}(\alpha_i \wedge c_i)$, where $c_i \in S$, $\alpha_i \in I$, $\max\{\alpha_i | i \in A\} = 1$.

For a capacity $c \in \underline{M}X$ and a number $\varepsilon > 0$ consider the closed subset

$$G_c = \big\{(c', \alpha) \mid c' \in S, \alpha \in I, \; \alpha \leq \max\big\{0, 1 - (\hat{d}(c, c') - \hat{d}(c, S))/\varepsilon\big\}\big\}$$

of $S \times I$.

Define a capacity $\tilde{c}_\varepsilon$ by the formula

$$\tilde{c}_\varepsilon = \bigvee_{i \in A} \{\alpha_i \wedge c_i | (c_i, \alpha_i) \in G_c\}.$$

Although the capacity $\tilde{c}_\varepsilon$ is not the closest to $c \in \underline{M}X$ in the subspace S, it is almost the closest in the following sense [6].

Theorem. *For a capacity $c \in \underline{M}X$, a number $\varepsilon > 0$ and a closed I-convex subspace $S \subset \underline{M}X$ the capacity $\tilde{c}_\varepsilon$ belongs to S and satisfies the inequality $\hat{d}(c, \tilde{c}_\varepsilon) \leq \hat{d}(c, S) + \varepsilon$. The mapping $\Phi : \underline{M}X \times (0, diam\underline{M}X] \to S \colon (c, \varepsilon) \mapsto \tilde{c}_\varepsilon$, is continuous.*

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Metrizability of compact topological pre-Clifford semigroups

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A semigroup is called *pre-Clifford* if it is a union of groups, and *Clifford* if it is an inverse pre-Clifford semigroup.

In the talk we shall discuss the following problem posed by B. M. Bokalo in the context of compact topological Clifford semigroups.

Problem. Is a compact topological pre-Clifford semigroup S metrizable if the set E_S of idempotents of S is metrizable and all subgroups of S are metrizable?

In [1] it was shown that for compact Clifford semigroups the answer to this problem is indepenedent of **ZFC**. We prove that the same is true for compact pre-Clifford semigroups.

Theorem. *A compact topological pre-Clifford semigroup S is metrizable if the set E_S of idempotents of S is metrizable, all subgroups of S are metrizable and one of the following conditions holds:*

1. $\mathbf{MA} + \neg\mathbf{CH}$ *holds;*
2. *for the Green relation* $\mathscr{J} = \{(x,y) \in S \times S \colon SxS = SyS\}$ *the compact topological semilattice* $S/\mathscr{J}$ *is Lawson;*
3. *the space* E_S *is zero-dimensional.*

Example (Banakh, 2003). Under **CH** there exists a non-metrizable compact inverse Clifford semigroup with metrizable semilattice of idempotents E_S and metrizable maximal subgroups.

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Lattices of uniformly continuous functions

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By examining relations between topological properties of a topological space X and algebraic properties of its set $C(X)$ of all continuous real-valued functions on X, lattice structure of the latter is usually used, sometimes explicitly, sometimes implicitly. Corresponding results for a substructure $U(X)$ of all uniformly continuous functions on a uniform space X should give more information for X but are more difficult to prove. We shall forget about other algebraic operations on $U(X)$ and use the lattice structure only.

The following problems are of some interest (all have satisfactory answers for $C(X)$):

- Which uniform spaces are determined by their lattices $U(X)$ – uniformly, proximally, topologically? (Banach-Stone–like theorems.) A modification of the question uses bounded functions only.
- If $U(X)$ and $U(Y)$ are lattice isomorphic, are $U^*(X)$ and $U^*(Y)$ lattice isomorphic, too? (Equivalently: are the Samuel compactifications of X, Y homeomorphic?)
- Characterize $U(X)$ among lattices, i.e., find out lattice properties of $U(X)$ such that a lattice has those properties iff it is isomorphic to the lattice $U(X)$ for some uniform space X.

Recent results and relations concerning the above problems will be discussed.

The zero-divisors graph of a matrix semigroup

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In a manner analogous to the commutative case, the zero-divisor graph of a non-commutative semigroup S can be defined as the directed graph $\Gamma(S)$ whose vertices are all non-zero zero-divisors of S in which for any two distinct vertices x and y, $x \to y$ is an edge if and only if $xy = 0$.

Initially, the concept of a zero-divisor graph firstly was introduced for the commutative rings (semigroups). Later this concept was extended to arbitrary rings [1–3] and to non-commutative semigroups [4, 5].

In the talk we shall discuss the interplay between the properties of a matrix semigroup S over a finite ring R and the graph-theoretic properties of $\Gamma(S)$, $\Gamma(R)$: the connectedness, diameter, existence of sources, sinks, etc.

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On Kuratowski partitions

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A function $f\colon X \to Y$ between topological spaces has *the Baire property* iff for each open set $V \subset Y$ the preimage $f^{-1}(V)$ has the Baire property in X (i.e., is open modulo a meager set in X).

According to a well-known theorem attributed to K. Kuratowski, if a function $f : X \to Y$ from a metrizable space X to a separable metrizable space Y has the Baire property, then for some meager set $M \subset X$ the restriction $f|X \setminus M$ is continuous.

In 1935 K. Kuratowski [2] raised a question whether the assumption of separability of Y is essential. The question is sensible if X fulfills Baire theorem.

In [3] Solovay considered partitions of the interval $[0, 1]$ into meager sets and using metamathematical methods showed that the union of suitable sets of the partition is non-measurable in the sense of category. Thus the Kuratowski's question proved to be equivalent to the existence of a partition $\mathcal{P}$ of a space X into meager subsets such that for every subfamily $\mathcal{P}' \subset \mathcal{P}$ the union $\bigcup \mathcal{P}'$ has the Baire property in X. Such partitions are called *Kuratowski partitions*.

It is provable that the existence of a Kuratowski partition of a Baire (complete) metric space is equiconsistent in **ZFC** with the existence of a measurable cardinal.

The number of papers concerning Kuratowski partitions is relatively small. The latest one is [1] in which the authors raised the problem of the existence of Kuratowski partitions of the Ellentuck space $[\omega]^\omega_{EL}$, (i.e. a space $[\omega]^\omega$ of infinite subsets of ω, equipped with the topology generated by the base consisting of the sets of the form $[a, A] = \{B \in [A]^\omega : a \subset B \subseteq a \cup A\}$, where $a \in [\omega]^{<\omega}$, $A \in [\omega]^\omega$). In the proof of the main result of [1] saying that no non-meager subspace of $[\omega]^\omega_{EL}$ admits a Kuratowski partition there is a gap but the theorem remains true. We will show the correct proof of this theorem. We will also show that Kuratowski partitions are strongly connected with the structure of quotient algebras and constructions of non-measurable sets.

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Extension of Baire-one functions from countable sets

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Let $\mathrm{B}_1(X)$ be the collection of all Baire-one functions on a topological space X. A subspace E of a topological space X is called

- B_1-*embedded* (B_1^*-*embedded*) in X if any (bounded) function $f \in \mathrm{B}_1(E)$ can be extended to $\bar{f} \in \mathrm{B}_1(X)$;
- 1-*embedded in* X if any functionally G_δ-set in E can be extended to a functionally G_δ-set in X;
- *ambiguously* 1-*embedded in* X if any functionally ambiguous set in E can be extended to a functionally ambiguous set in X;
- *well* 1-*embedded in* X, if for any functionally G_δ-set $A \subseteq X$ disjoint with E there exists a function $f \in \mathrm{B}_1(X)$ such that $E \subseteq f^{-1}(0)$ and $A \subseteq f^{-1}(1)$.

We show that a subspace E of a topological space X is B_1^*-embedded in X if and only if E is ambiguously 1-embedded in X. We prove that E is B_1-embedded in X if and only if E is 1-embedded and well 1-embedded in X.

Moreover, any countable hereditarily irresolvable completely regular space is B_1^*-embedded in βX but not B_1-embedded in βX.

On finite sums of periodic functions

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Every function f acting from the real line $\mathbb{R}$ into itself is representable as a series of periodic functions, which is uniformly convergent to f on any bounded subinterval of $\mathbb{R}$. In particular, f is a pointwise limit of a sequence of finite sums of periodic functions, and those sums converge uniformly to f on any bounded subinterval of $\mathbb{R}$.

The above-mentioned result is effective in the sense that it does not rely on the Axiom of Choice. So, if the function f has a nice descriptive structure (e.g., f is Borel or Lebesgue measurable or possesses the Baire property), then the same structure is preserved by all corresponding periodic functions.

There exists a real-valued analytic function g on $\mathbb{R}$ which cannot be represented as the limit of a uniformly convergent sequence of finite sums of periodic functions. On the other hand, by using the technique of Hamel bases and independent families of sets, g can be expressed as the limit of a pointwise convergent sequence of finite sums of periodic functions on $\mathbb{R}$, this sequence converges uniformly to g on any bounded subinterval of $\mathbb{R}$, and all periodic functions in the sums have periods belonging to the line segment.

On the conformal and geodesic mappings of quasi-Einstein spaces

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We study conformal and geodesic mappings of quasi-Einstein spaces and prove that they are closed with respect to circular mappings. We introduce the concept of mobility of pseudo-Riemannian spaces with respect to concircular mappings. We have found the tensor characteristic of maximal mobility for quasi-Einstein spaces. We have proved that quasi-Einstein pseudo-Riemannian space with constant scalar curvature is closed relatively to nontrivial geodesic mappings.

The Banach-Mazur game for graphs

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Extending partial representations of trapezoid graphs

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A trapezoid graph is an intersection graph of trapezoids spanned between two horizontal lines. The partial representation extension problem for trapezoid graphs is a generalization of the recognition problem: given a graph G and an assignment ϕ of trapezoids to some vertices of G, decide whether ϕ can be extended to an intersection model of the entire graph G. We prove that this problem can be decided in polynomial time. This way, we solve the partial representation

extension problem for one of the two major remaining classes of geometric intersection graphs (circular-arc graphs being the other), for which recognition is decidable in polynomial time, but the complexity of partial representation extension has been unknown. As a corollary, we also provide a polynomial-time algorithm for partial representation extension of co-bipartite circular-arc graphs.

Deformations of smooth functions on compact surfaces

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Let M be a compact surface, P be either $\mathbb{R}$ or S^1, and $f : M \to P$ be a smooth map. Denote by $\mathcal{S}(f)$ the group of diffeomorphisms $h : M \to M$ preserving f, i.e. $f \circ h = h$, and let $\mathcal{S}'(f)$ be its subgroup consisting of diffeomorphisms isotopic to the identity via isotopies that are not necessarily f-preserving. The groups $\pi_0\mathcal{S}(f)$ and $\pi_0\mathcal{S}'(f)$ can be regarded as analogues of mapping class group for f-preserving diffeomorphisms. The aim of the talk is to describe precise algebraic structure of the group $\pi_0\mathcal{S}'(f)$ and some of its subgroups and quotients for a large class of smooth maps $f : M \to P$ containing all Morse maps, where M is orientable and distinct from 2-sphere. In particular it is shown that $\pi_0\mathcal{S}'(f)$ is solvable.

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On semitopological interassociates of the bicyclic monoid

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We shall follow the terminology of [2,3,5]. The bicyclic monoid $\mathscr{C}(p,q)$ is the semigroup with the identity 1 generated by two elements p and q subjected only to the condition $pq = 1$. An interassociate of a semigroup $(S,\cdot)$ is a semigroup $(S,*)$ such that for all $a,b,c \in S$, $a\cdot(b*c) = (a\cdot b)*c$ and $a*(b\cdot c) = (a*b)\cdot c$.

This definition of interassociativity was studied extensively in 1996 by Boyd et al [1]. Certain classes of semigroups are known to give rise to interassociates with various properties. For example, it is very easy to show that if S is a monoid, every interassociate must satisfy the condition $a * b = acb$ for some fixed element $c \in S$ (see [1]). In the paper [4] the bicyclic monoid $\mathscr{C}(p,q)$ and its interassociates are investigated. In particular, if p and q are the generators of the bicyclic semigroup $\mathscr{C}(p,q)$ and m and n are fixed nonnegative integers, the operation $a *_{m,n} b = aq^m p^n b$ is known to be an interassociate and such associate of $\mathscr{C}(p,q)$ is denoted by $\mathscr{C}_{m,n}$. We study Hausdorff topologizations of the interassociate $\mathscr{C}_{m,n}$ of the bicyclic monoid as a semitopological semigroup.

Theorem 1. *For arbitrary non-negative integers m and n every Hausdorff topology τ turning $\mathcal{C}_{m,n}$ into a semitopological semigroup is discrete. Thus $\mathscr{C}_{m,n}$ is a discrete subspace of any topological semigroup containing it.*

Theorem 2. *If m and n are arbitrary non-negative integers the interassociate $\mathscr{C}_{m,n}$ of the bicyclic monoid $\mathscr{C}(p,q)$ is a dense subsemigroup of a Hausdorff semitopological semigroup $(S,\cdot)$ and $I = S \setminus \mathscr{C}_{m,n} \neq \varnothing$, then I is a two-sided ideal of the semigroup S.*

For arbitrary non-negative integers m, m by $\mathscr{C}^0_{m,n}$ we denote the interassociate $\mathscr{C}_{m,n}$ with an adjoined zero 0 of the bicyclic monoid $\mathscr{C}(p,q)$, i.e., $\mathscr{C}^0_{m,n} = \mathscr{C}_{m,n} \sqcup \{0\}$.

Theorem 3. *Let m and n be arbitrary non-negative integers. If $\mathscr{C}^0_{m,n}$ is a Hausdorff locally compact semitopological semigroup, then either $\mathscr{C}^0_{m,n}$ is discrete or compact.*

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On weak and pointwise topologies in function spaces

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For a compact space K we denote by $C_w(K)$ ($C_p(K)$) the space of continuous real-valued functions on K endowed with the weak (pointwise) topology. We will present results and open questions related to the following general problem: *Let K and L be infinite compact spaces. Can it happen that $C_w(K)$ and $C_p(L)$ are homeomorphic?*

M. Krupski proved that the above problem has a negative answer when $K = L$ and K is finite-dimensional and metrizable. We extend this result to the class of finite-dimensional Valdivia compact spaces K.

This is a joint research with Mikołaj Krupski.

$\mathbb{Q}_\infty^*$-representation of real numbers determined by an infinite double stochastic matrices and sets associated with them

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We consider the Q_∞^*-representation of the fractional part of a real number, which is a number encoding with infinite alphabet under condition that it is determined by a doubly-stochastic matrix depending on one parameter. Topological, metric properties of real number sets with restriction on using symbols in the Q_∞^*-representation are investigated.

Kuratowski monoids of n-topological spaces

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Generalizing the famous 14-set closure-complement Theorem of Kuratowski from 1922, we prove that for a set X endowed with n pairwise comparable topologies $\tau_1 \subset \cdots \subset \tau_n$, by repeated application of the operations of complement and closure in the topologies $\tau_1, \dots, \tau_n$ to a subset $A \subset X$ we can obtain at most $2K(n) = 2\sum_{i,j=0}^{n} \binom{i+j}{i}\binom{i+j}{j}$ different sets.

For $n \le 7$ the numbers $2K(n)$ are equal to: **1**4, 126, 1394, 17098, 222066, 2991359, 41334926, 582040566, 8315731286.

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Contribution of K. Kuratowski to the theory of separately continuous functions

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R. Baire [1,2] proposed a method of classification of functions of a real variable that now is referred to as Baire's classification. H. Lebesgue (who observed in his first published work [3] that each separately continuous function $f\colon \mathbb{R}^2 \to \mathbb{R}$ belongs to the first Baire class) in [4] proposed another classification of functions $f\colon \mathbb{R}^n \to \mathbb{R}$, known as the Lebesgue classification (sometimes wrongly associated with the name of E. Borel). Baire's and Lebesgue's classifications can be generalized to functions beteween general topological spaces. Many mathematicians contributed to this investigation, in particular, F. Hausdorff.

For topological spaces X, Y and a countable ordinal number α by $B_\alpha(X,Y)$ and $H_\alpha(X,Y)$ we denote the set of all functions $f\colon X \to Y$ of the Baire or Lebesgue class α, respectively. According to the classical Lebesgue-Hausdorff's theorem and its generalizations, these two classes are closely related.

For a function $f\colon X \times Y \to Z$ and a point $p = (x,y) \in X \times Y$ we put

$$f^x(y) = f(p) = f_y(x).$$

For a property (P) of maps by $P(X,Y)$ we denote the set of all functions $f\colon X \to Y$ that have the property (P). For a function $f\colon X\times Y \to Z$ and a property (P) we put

$$X_P(f) = \{x \in X\colon f^x \in P(Y,Z)\} \qquad \text{and} \qquad Y_P(f) = \{y \in y\colon f_y \in P(X,Z)\}.$$

For properties P and Q, we introduce the classes

$$PQ(X\times Y, Z) = \left\{f \in Z^{X\times Y}\colon X_Q(f) = X \text{ and } Y_P(f) = Y\right\}$$

and

$$P\bar{Q}(X\times Y, Z) = \left\{f \in Z^{X\times Y}\colon X_Q(f) \text{ is dense in } X \text{ and } Y_P(f) = Y\right\}.$$

Thus, for topological spaces X, Y and Z, we shall consider the classes $CC(X\times Y,Z)$
of separately continuous functions $f\colon X\times Y\to Z$ and its analogues: the classes $CB_\alpha(X\times Y,Z)$, $CH_\alpha(X\times Y,Z)$, etc.

Lebesgue's theorem on the inclusion $CC(\mathbb{R}^2,\mathbb{R}) \subseteq B_1(\mathbb{R}^2,\mathbb{R})$ was significantly developped in the works of H. Hahn [5, §39], W. Rudin [6] and many other mathematicians. Rudin was the first who applied to this problem the Stone Theorem on the paracompactness of metrizable space and partitions of unity. This development continues today (see, for example, [7]). This concerns theorems on the inclusions $CB_\alpha(X\times Y,Z) \subseteq B_{\alpha+1}(X\times Y,Z)$ or $C\bar{B}_\alpha(X\times Y,Z) \subseteq B_{\alpha+1}(X\times Y,Z)$.

K. Kuratowski was the first who found [8] that $CH_\alpha(X\times Y,Z) \subseteq H_{\alpha+1}(X\times Y,Z)$ if spaces X, Y and Z are metrizable and X is separable (see also [9, p. 387]). In fact, with this conditions on X the inclusion $C\bar{H}_\alpha(X\times Y,Z) \subseteq H_{\alpha+1}(X\times Y,Z)$ holds for any perfect space Y and perfectly normal space Z.

A bit later this result of Kuratowski was improved by D.Montgomery [10] and K. Kuratowski himself [11] who removed the condition of separability of the space X (see also [9, p. 388]). The so-called Montgomery's $\mathcal{M}$-operation was used in the proof [9, p. 267]. This operation is also used in the proof that every set A in a metrizable space X, that locally belongs to the additive class α or multiplicative Borel class $\alpha > 0$ is a set in the same class α [9, p. 266]. Another proof of this proposition, based on the Stone theorem on the paracompactness of metrizable spaces, was found by Michael [12] and Nagami [13] (see. [9, p. 367]). It turns out that the Kuratowski-Montgomery result can also be proved ina general form using Stone theorem [14, Theorem 2]: if X is metrizable, Y is perfect and Z is perfectly normal, then $C\bar{H}_\alpha(X\times Y,Z) \subseteq H_{\alpha+1}(X\times Y,Z)$. This result was further developed in the papers of Karlova and Sobchuk.

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The development of Hahn's theorem on an intermediate function

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By the Hahn theorem about intermediate function [1], for a metrizable space X and respectively upper and lower semicontinuous functions $g \leq h$ on X there exists a continuous function $f\colon X \to \mathbb{R}$ such that $g \leq f \leq h$. Later this theorem was generalized [2, 3] to normal spaces and it was found that normality is necessary for its implementation. In [3–5] some analogues of Hahn's theorem were obtained, where an intermediate function satisfies the strict inequality $g(x) < f(x) < h(x)$ on X if $g(x) < h(x)$ on X or the equity $g(x) = f(x) = h(x)$ if $g(x) = h(x)$ and the strict inequality $g(x) < f(x) < h(x)$ if $g(x) < h(x)$.

In recent years, counterparts of Hahn's theorem were proved for monotone [6] or convex [7] functions. In particular, in [6] it was proved that for respectively upper and lower semicontinuous increasing functions $g, h\colon [a,b] \to \mathbb{R}$ with $g \le h$ there is a continuous increasing function f such that $g \le f \le h$. A new question appeares here.

Question. Let $g, h\colon [a,b] \to \mathbb{R}$ be functions of bounded variation such that g is upper semicontinuous, h lower semicontinuous and $g \le h$. Is there a continuous function $f\colon [a,b] \to \mathbb{R}$ of bounded variation such that $g \le f \le h$?

So far, the answer to this question is not found yet.

But there were related questions about the existence of intermediate piecewise linear or infinitely differentiable functions. Using one of methods from the proof of Heine-Borel's lemma or Heine-Borel's lemma itself, we managed to obtain the following results.

We will call a pair (g, h) of functions $g, h\colon X \to \mathbb{R}$ a *Hahn* (*strict*) *pair* on X if g is upper semicontinuous, h is lower semicontinuous, and $g(x) \le h(x)$ (resp. $g(x) < h(x)$) on X.

Theorem 1. *Let (g, h) be a Hahn strict pair on the segment* $[a,b]$*. Then there is a piecewise linear function* $f\colon [a,b] \to \mathbb{R}$*, such that* $g(x) < f(x) < h(x)$ *on* $[a,b]$*.*

Theorem 2. *Let (g, h) be a Hahn strict pair on the segment* $[a,b]$*. Then there exists an infinitely differentiable function* $f\colon [a,b] \to \mathbb{R}$ *such that* $g(x) < f(x) < h(x)$ *on* $[a,b]$*.*

The main observation is that upper and lower semicontinuous functions $g, h\colon X \to \mathbb{R}$ for which $g(x_0) < h(x_0)$ are locally strictly separated by arbitrary constant γ, for which $g(x_0) < \gamma < h(x_0)$. Using the compactness of the segment, we come to a finite coverings which allows to build the required intermediate function.

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Cones of locally connected curves

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It is well known that cones of non-homeomorphic spaces can be homeomorphic (e.g. Cone(S^1) and Cone(I)). In my talk, I will show that if X and Y are locally connected curves not being ANR then Cone(X) and Cone(Y) are homeomorphic if and only if X and Y are homeomorphic.

Continuous extension from countable sets

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For a topological space X by $C_p(X)$ (resp. $C_p^*(X)$) we denote the space of all (bounded) real-valued continuous functions on X, endowed with the topology of pointwise convergence.

The following questions were posed in [1] (see also Questions 4.10.3 and 4.10.11 in [2]).

Question 1. Let X be a pseudocompact space such that for any countable set $A \subseteq X$ there exists a (linear) continuous operator $\varphi : C_p^*(A) \to C_p(X)$ with $\varphi(y)|_A = y$ for every $y \in C_p(A)$. Must X be finite?

Question 2. Let X be the subspace of all weak P-points of $\beta\omega \setminus \omega$. Is it true that for any countable set $A \subseteq X$ there exists a (linear) continuous operator $\varphi : C_p^*(A) \to C_p(X)$ such that $\varphi(y)|_A = y$ for every $y \in C_p(A)$?

A subset A of a topological space X is *strongly functionally discrete in* X if there exists a discrete family $(G_a)_{a\in A}$ of functionally open sets $G_a \ni a$ in X.

Theorem 1. *Let X be a topological space and $A \subseteq X$ be a discrete countable subspace of X. Then the following conditions are equivalent:*

1. *there exists a (linear) continuous mapping $\varphi : C_p^*(A) \to C_p^*(X)$ such that $\varphi(y)|_A = y$ for every $y \in C_p^*(A)$;*
2. *there exists a (linear) continuous mapping $\varphi : C_p(A) \to C_p(X)$ such that $\varphi(y)|_A = y$ for every $y \in C_p(A)$;*

(3) *there exists a (linear) continuous mapping* $\varphi : C_p^*(A) \to C_p(X)$ *such that* $\varphi(y)|_A = y$ *for every* $y \in C_p^*(A)$*;*
(4) *the set* A *is strongly functionally discrete in* X*.*

Theorem 2. *Let* X *be a topological space such that for every countable set* $A \subseteq X$ *there exists a continuous mapping* $\varphi : C_p^*(A) \to C_p(X)$ *with* $\varphi(y)|_A = y$ *for every* $y \in C_p^*(A)$*. Then*

(1) *every discrete countable subspace* A *of* X *is a strongly functionally discrete set in* X*;*
(2) *if* X *is a* T_2*-space in which every locally finite system of functionally open sets is finite (in particular, if* X *is a pseudocompact) then* X *is finite;*
(3) X *does not equal to the space of all weak* P*-points in* $\beta\omega \setminus \omega$*;*
(4) *if* X *is a completely regular space, then every countable subspace* A *of* X *is a (strongly functionally) discrete set in* X*.*

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On stability of asymptotic property C

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We show that Dranishnikov's asymptotic property C is preserved by the direct product and free products of metric spaces. In particular, if G and H are groups with asymptotic property C, then both $G \times H$ and $G * H$ have asymptotic property C. We also prove that a group G has asymptotic property C if G contains a normal subgroup H of finite asymptotic dimenion such that the quotient group G/H has asymptotic property C. The groups are assumed to have left-invariant proper metrics and need not be finitely generated. These results resolve questions of Dydak and Virk, of Bell and Moran, and an open problem in topology from the Lviv Topological Seminar.

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Localization in graphs with colored edges

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We consider a game played by two players – Cop and Robber. A board for the game is some fixed graph $G = (V, E)$. At the beginning Cop colors the edges of G with k fixed colors (she need not obey any rules). Then Cop picks some r vertices, we can imagine that these vertices are checked, each by one cop, and Robber pick one vertex, say v, which can be viewed as a position of Robber on G and gives to Cop the information of all shortest paths from v to all picked by her vertices. This information consists of a sequence of colors which appear on a given path. If Cop can guess where Robber is, i.e. v is the unique vertex of G with such sequences of colors on the shortest paths from it to the vertices picked by Cop, she wins. If not, game continues. Cop picks r vertices of G and Robber can stay in v or move to any of the neighbors of v and then gives information of the colors on the shortest paths form her position to the vertices picked by Cop. And so on. If Cop is able to localize Robber after finite number of turns, then Cop wins the game. If Robber is able to evade Cop eternally, then Robber wins the game. We study the numbers of colors and cops which allow Cop to win on some classes of graphs.

Equi-cliquishness and the Hahn property

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Investigations of the relationships between separate and joint continuity, which traces its history back to classical works of Baire and Osgood, were continued in papers of many mathematicians of the twentieth century. One of directions of this research deals with the question of whether a mapping $f : X \times Y \to Z$ has the Hahn property, i.e. there is a residual subset A of X such that $A \times Y \subseteq C(f)$ where $C(f)$ is the set of continuity points of f.

We shall survey the existing results and present some new necessary and sufficient conditions for the Hahn property of a given function of two variables.

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Topological fractals

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We deal with the topological point of view on fractals. One of the simplest methods of constructing self-similar fractals are Iterated Function Systems (IFS). This is a finite family of contractive self-maps on a metric space. A compact subset of such space, which is strongly invariant by such family, is called an IFS-*attractor*. Such sets often have a fractal structure (e.g. Sierpinski triangle, Koch curve), so we called them *fractals*.

Some of compact spaces become an IFS-attractors after a suitable topological transformation. Some fractals can be also transformed so that they are no longer IFS-attractors. During the lecture we present various examples of such spaces.

Considering a topological version of contractive maps we obtain a topological IFS and its attractor called a topological fractal. Being a topological fractal is a topological invariant. We expose some conditions for compact spaces (especially Peano continua) which are sufficient for being a topological fractal.

Spherical sets avoiding a prescribed set of angles

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Witsenhausen in 1974 asked about the supremum of the measure of a subset of the n-dimensional unit sphere not containing a pair of orthogonal vectors. I present some recent results, joint with Evan DeCorte. We show that the supremum is attained by using methods of harmonic analysis. Then we concentrate on the first open case $n = 2$, where graph theory methods turned out to be useful.

Paper reference:
http://imrn.oxfordjournals.org/content/early/2015/11/29/imrn.rnv319

On the monoid of monotone injective partial selfmaps of $\mathbb{N} \times \mathbb{N}$ with cofinite domains and images

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Let $\mathbb{N}$ be the set of natural number and $\mathbb{N}^2_{\leqslant}$ be the set $\mathbb{N}^2$ with the partial order $\leqslant$ defined by $(x, y) \leqslant (x', y')$ iff $x \leq x'$ and $y \leq y'$.

By $\mathscr{PO}_\infty(\mathbb{N}^2_{\leqslant})$ we denoted the semigroup of monotone injective partial selfmaps of $\mathbb{N}^2_{\leqslant}$ having cofinite domain and range.

We describe properties of elements of the semigroup $\mathscr{PO}_\infty(\mathbb{N}^2_{\leqslant})$ as monotone partial bijection of $\mathbb{N}^2_{\leqslant}$ and show that the group of units of $\mathscr{PO}_\infty(\mathbb{N}^2_{\leqslant})$ is isomorphic to the cyclic group of the order two. Also we describe the subsemigroup of idempotents of $\mathscr{PO}_\infty(\mathbb{N}^2_{\leqslant})$ and the Green relations on $\mathscr{PO}_\infty(\mathbb{N}^2_{\leqslant})$. In particularly we show that $\mathscr{D} = \mathscr{J}$ in $\mathscr{PO}_\infty(\mathbb{N}^2_{\leqslant})$.

Paper reference: **http://arxiv.org/abs/1602.06593**

Topology of flows with a fixed point on the boundary of 2-manifold

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We consider flows on a connected compact 2-manifold with connected boundary, which have exactly one fixed point on the boundary and doesn't have closed orbits.

Separatrixes divide enough small neighborhood of the fixed point into parts, which are called *angles.*

There are three possible cases of angles:

1) hyperbolic angle;
2) elliptic angle;
3) parabolic angle.

The case of parabolic angle has two subcases:

a) angle-source;
b) angle-sink.

We show that all separatrixes divide the 2-manifold into regions, which have one angle-source and one angle-sink or single parabolic angle, and other angles are hyperbolic.

We will construct complete topological invariants of the flow.

Hamilton operators and related integrable differential-algebraic Novikov-Leibniz structures

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As it is well known, many of integrable Hamiltonian systems, discovered during the last decades, were understood owing to the Lie-algebraic properties of their internal hidden symmetry structures. A first account of the Hamiltonian operators and related differential-algebraic structures, lying in the background of integrable systems was given by I. Gelfand and I. Dorfman and later was extended by B. Dubrovin and S. Novikov, and also by A. Balinski and S. Novikov. There were also devised some new special differential-algebraic techniques for studying the Lax type integrability and the structure of related Hamiltonian operators for a wide class of the Riemann type hydrodynamic hierarchies. Just recently a lot of works appeared being devoted to the finite dimensional representations of the Novikov algebra. Their importance for constructing integrable multi-component nonlinear Camassa-Holm type dynamical systems on functional manifolds was demonstrated by I. Strachan and B. Szablikowski, where there was suggested in part the Lie-algebraic imbedding of the Novikov algebra into the general Lie-Poisson orbits scheme of classification Lax type integrable Hamiltonian systems. It is also worth of mentioning the related work by D. Holm-R. Ivanov where there were also constructed integrable multi-component nonlinear Camassa-Holm type dynamical systems on functional manifolds.

In our work we succeeded in formal differential-algebraic reformulating the classical Lie algebraic scheme and developed an effective approach to classification of the algebraic structures lying in the background of the integrable multicomponent Hamiltonian systems. In particular, we have devised a simple algorithm allowing to construct new algebraic structures within which the corresponding Hamiltonian operators exist and generate integrable multicomponent dynamical systems. We show, as examples, that the well known Novikov algebraic structure, obtained before as a condition for a matrix differential expression to be Hamiltonian and as that on a flat torsion free left-invariant affine connection on affine manifolds, affine structures and convex homogeneous cones, appears within the devised approach as a differentiation on the Lie-algebra, naturally associated with a suitably constructed differential loop algebra. As a direct generalization of this example it is obtained two new differentiations, whose background algebraic structures coincide, respectively, with the well known right Leibniz algebra, introduced in, and with a new so-called non-associative “Riemann” algebra, which naturally generate different Hamiltonian operators, describing a wide class of multi-component hierarchies of integrable multi-component hydrodynamic Riemann type systems. Their reductions appeared to be closely related both with the mentioned above integrable Camassa-Holm and with the Degasperis-Processi dynamical systems, and are of special interest from the equivalence transformation point of view, devised recently. A classical Poisson manifold approach, closely related with that analyzed in the work and allowing effectively enough to construct Hamiltonian operators, is also briefly revisited.

Kazimierz Kuratowski and his Lviv students

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Kazimierz Kuratowski was a professor and the head of the third department of mathematics on the “general” faculty in the Lviv Polytechnics from 1927 to 1933. The "general" faculty was established in 1921/22 academic year. On this faculty studies were divided into three groups: mathematics, physics and chemistry and graphics. Students-mathematicians could specialize in the fields of: applied mathematics, pure mathematics and descriptive geometry. The students of this faculty attended lectures of Stefan Banach, Włodzimierz Stożek, Stanisław Ruziewicz, Antoni Łomnicki, Stefan Kaczmarz, Władysław Nikliborc and others. Among the students of K. Kuratowski there were Stanisław Ulam, Edward Otto, Tadeusz Posament and others. After the abolition of the faculty and the department, K. Kuratowski came back to Warsaw where he obtained the position of professor in the Warsaw University.

Max-Plus convex compacta: a categorical approach

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Let $I(X)$ denote the set of all idempotent (Maslov) probability measures on a compactum X. We consider $I(X)$ as a subspace of $\prod_{\phi\in C(X)}[\min\phi,\max\phi]$. The construction I is a functor which can be completed to a monad $\mathbb{I}$, see [1].

M. Zaricznyi proved that each compact Max-Plus convex subset in $\mathbb{R}^n$ with the idempotent barycenter map is an $\mathbb{I}$-algebra and posed a question of characterizing the category of $\mathbb{I}$-algebras. We solve this question showing that each $\mathbb{I}$-algebra is isomorphic to a compact Max-Plus convex subset in a Tychonov power $\mathbb{R}^\tau$ with the idempotent barycenter map and each morphism of $\mathbb{I}$-algebras is a Max-Plus affine map.

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Nonmeasurable images in Polish space with respect to selected sigma ideals

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We present results on nonmeasurability (with respect to a selected σ-ideal on a Polish space) of images of functions defined on Polish spaces. In particular, we give a positive answer to the following question: *Is there a subset of the unit disc in the real plane such that continuum many projections onto lines are Lebesgue measurable and continuum many projections are not?*

Minimality of the semidirect product

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A topological group is minimal if it does not admit a strictly coarser Hausdorff group topology. We prove that for a compact topological group G, the semidirect product $G \leftthreetimes P$ is minimal for every closed subgroup P of $\mathrm{Aut}(G)$.

In general, the compactness of G is essential; $G \leftthreetimes P$ might be nonminimal even for precompact minimal groups G as it follows from an example of Eberhardt-Dierolf-Schwanengel. Some of the results were inspired by a work of Gamarnik.

Paper reference: **http://arxiv.org/abs/1511.07021v2**

Interplay between dimensions of micro- and macro-fractals

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In the talk we shall discuss the interplay between the micro-dimension of a fractal generated by a contracting multi-valued map Φ on a complete metric space X and the macro-dimension of the macro-fractal generated by the inverse multi-valued map Φ^{-1}. We find conditions on X and Φ guaranteeing that the micro-dimension of the fractal coincides with the macro-dimension of the dual macro-fractal.

Discrete subsets in topological groups and countable extremally disconnected groups

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The main results of this talk stem from attempts to solve the following problem of Arhangel'skii.

Problem (Arhangel'skii, 1967)**.** Does there exist in **ZFC** a nondiscrete Hausdorff extremally disconnected topological group?

Recall that a topological space is said to be *extremally disconnected* if the closure of any open set in this space is open (or, equivalently, the closures of any two disjoint open sets are disjoint).

In the talk, a solution of Arhagel'skii's problem for countable groups is presented. Namely, it is proved that *the nonexistence of a countable nondiscrete Hausdorff extremally disconnected group is consistent with* **ZFC**. The proof of this assertion is based on the following two main theorems.

Theorem 1. *Any countable nondiscrete topological group whose identity element has nonrapid filter of neighborhoods contains a discrete sequence with precisely one limit point.*

Theorem 2. *If there are no rapid filters, then any countable nondiscrete Hausdorff Boolean topological group contains two disjoint discrete subsets for each of which the zero of the group is a unique limit point.*

(A filter $\mathcal{F}$ on ω is said to be *rapid* if every function $\omega \to \omega$ is majorized by the increasing enumeration of some element of $\mathcal{F}$. The nonexistence of rapid filters is consistent with **ZFC**.)

These two theorems have a number of consequences concerning contable topological groups with extremal properties. For example, they imply that if there are no rapid ultrafilters, then any countable nondiscrete topological group is ω-resolvable (i.e., can be partitioned into countably many dense subsets) and that the nonexistence of countable nondiscrete nodec groups is consistent with **ZFC** (*nodec* means that all nowhere dense sets are closed).

Paper reference: **http://arxiv.org/abs/1608.03546**

Eliminating Higher-Multiplicity Intersections, III. Codimension 2

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We study conditions under which a finite simplicial complex K can be mapped to $\mathbb{R}^d$ without higher-multiplicity intersections. An *almost r-embedding* is a map $f\colon K \to \mathbb{R}^d$ such that the images of any r pairwise disjoint simplices of K do not have a common point. We show that if r is not a prime power and $d \geq 2r+1$, then there is a counterexample to the topological Tverberg conjecture, i.e., there is an almost r-embedding of the $(d+1)(r-1)$-simplex in $\mathbb{R}^d$. This improves on previous constructions of counterexamples (for $d \geq 3r$) based on a series of papers by M. Özaydin, M. Gromov, P. Blagojević, F. Frick, G. Ziegler, and the second and fourth present author.

The counterexamples are obtained by proving the following algebraic criterion in codimension 2: If $r \geq 3$ and if K is a finite $2(r-1)$-complex, then there exists an almost r-embedding $K \to \mathbb{R}^{2r}$ if and only if there exists a general position PL map $f\colon K \to \mathbb{R}^{2r}$ such that the algebraic intersection number of the f-images of any r pairwise disjoint simplices of K is zero. This result can be restated in terms of cohomological obstructions or equivariant maps, and extends an analogous codimension 3 criterion by the second and fourth author.

It follows from work of M. Freedman, V. Krushkal, and P. Teichner that the analogous criterion for $r = 2$ is false. We prove a beautiful lemma on singular higher-dimensional Borromean rings, yielding an elementary proof of the counterexample and the following result. For each (d, n) such that $d = \frac{3n}{2} + 1 \geq 4$ the algorithmic problem of recognition almost 2-embeddability of finite n-dimensional complexes in $\mathbb{R}^d$ is NP hard.

As another application of our methods, we classify ornaments $f\colon S^3 \sqcup S^3 \sqcup S^3 \to \mathbb{R}^5$ up to ornament concordance.

Paper reference: **http://arxiv.org/abs/1511.03501**

Feebly compact topologies on the semilattice $\exp_n \lambda$

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We discuss on feebly compact topologies τ on the semilattice $(\exp_n \lambda, \cap)$ such that $(\exp_n \lambda, \tau)$ is a semitopological semilattice. All compact semilattice topologies on $\exp_n \lambda$ are described. Also we proved that for an arbitrary positive integer n and an arbitrary infinite cardinal λ for a T_1-topology τ on $\exp_n \lambda$ the following conditions are equivalent:

(*i*) $(\exp_n \lambda, \tau)$ is a compact topological semilattice;
(*ii*) $(\exp_n \lambda, \tau)$ is a countably compact topological semilattice;
(*iii*) $(\exp_n \lambda, \tau)$ is a feebly compact topological semilattice;
(*iv*) $(\exp_n \lambda, \tau)$ is a compact semitopological semilattice;
(*v*) $(\exp_n \lambda, \tau)$ is a countably compact semitopological semilattice.

We construct a countably pracompact H-closed quasiregular non-semiregular topology τ_{fc}^2 such that $\left(\exp_2 \lambda, \tau_{\mathsf{fc}}^2\right)$ is a semitopological semilattice with discontinuous semilattice operation and prove that for an arbitrary positive integer n and an arbitrary infinite cardinal λ a semiregular semitopological semilattice $\exp_n \lambda$ is a compact topological semilattice.

Paper reference: **http://arxiv.org/abs/1606.00395**

Homeotopy groups of non-singular foliations on the plane

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We consider non-singular oriented foliations F on non-compact surfaces Σ whose spaces of leaves have structure similar to rooted trees of finite diameter. The interior of Σ is homeomorphic to $\mathbb{R}^2$. Denote by $\mathfrak{F}$ the class of such surfaces Σ.

Let $H^+(F)$ be the group of all homeomorphisms of Σ which maps leaves onto leaves and preserves their orientation. Let also K be the group of homeomorphisms of the quotient-space Σ/F. Denote by $H_0^+(F)$ and K_0 the corresponding subgroups consisting of homeomorphisms isotopic to identity mappings.

We describe the algebraic structure of the mapping class group $\pi_0 H^+(F) = H^+(F)/H_0^+(F)$ and establish the isomorphism between homeotopy groups $\pi_0 H^+(F)$ and $\pi_0 K = K/K_0$ under assumption that F is a foliation of some surface $\Sigma \in \mathfrak{F}$.

Paper reference: **http://arxiv.org/abs/1607.04097v1**

On certain uniformly open multilinear mappings

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Assume that X and Y are metric spaces. A mapping $T\colon X \to Y$ is called *open* if it sends every open set in X to an open set in Y. Equivalently, f is open if

$$\forall_{x\in X}\ \forall_{\varepsilon>0}\ \exists_{\delta>0}\ B(T(x),\delta) \subset T[B(x,\varepsilon)]$$

(here $B(z,r)$ stands for the open ball with center z and radius $r>0$ in a given space).

In [2], T is called *uniformly open* if T it satisfies a uniform version of the above condition, that is

$$\forall_{\varepsilon>0}\ \exists_{\delta>0}\ \forall_{x\in X}\ B(T(x),\delta) \subset T[B(x,\varepsilon)]$$

The classical Banach open mapping principle states that every linear continuous surjection between two Banach spaces is an open mapping. In fact, it is uniformly open since its openness at the origin implies the openness at the remaining points with the same δ. It is known that the counterpart of this principle for bilinear continuous surjections is false.

Pointwise multiplication is a natural example of a bilinear continuous surjective operator for several function spaces X. However, in general it need not be open. A well-known counterexample is the Banach algebra $C[0,1]$ of real-valued continuous functions on $[0,1]$ endowed with the supremum norm (see [3]).

During the talk I will present two results stating the uniform openness of bilinear operators and multilinear functionals. The first result deals with Banach spaces L^p and pointwise multiplication from $L^p \times L^q$ to L^r (where $1/p + 1/q = 1/r$). The second result is concerned with the nontrivial n-linear functionals on the product $X_1 \times \cdots \times X_n$ of normed spaces.

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Uniform powers of compacta and the proximal game

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The countable uniform power (or uniform box product) of a uniform space X is a special topology on X^ω that lies between the Tychonoff topology and the box topology. We solve an open problem posed by P. Nyikos showing that if X is a compact proximal space then the countable uniform power of X is also proximal (although it is not compact). By recent results of J. R. Bell and G. Gruenhage this implies that the countable uniform power of a Corson compactum is collectionwise normal, countably paracompact and Fréchet-Urysohn. We also give some results about first countability, realcompactness in countable uniform powers of compact spaces and explore questions by P. Nyikos about semi-proximal spaces.

Paper reference: **http://arxiv.org/abs/1409.4844**

Productively (and non-productively) Menger spaces

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A topological space is Menger if, for every sequence of open covers, we can produce a new cover by chosing finitely many open sets from each of the given covers. Menger's property is strictly stronger than being Lindelöf. Every σ-compact space is Menger, and even productively so: Every product of a σ-compact space and a Menger space is Menger.

Based on weak set theoretic hypotheses, we construct, in a purely combinatorial way, Menger sets of real numbers whose product is not Menger.

The Hurewicz property is a strong form of Menger's property. Using our method, we prove, assuming a portion of **CH**, that every productively Menger space is productively Hurewicz, and that the converse implication is not provable.

Selected mathematical achievements of Kazimierz Kuratowski

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Selected outstanding mathematical achievements of Kazimierz Kuratowski will be recalled.

Periodic Fibonacci words

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Let $F_n^*(m)$ denote the n-th member of the sequence of integers $F_n \equiv F_{n-1} + F_{n-2} \pmod{m}$, for all integer $n > 1$, and with initial values $F_0 = 0$ and $F_1 = 1$. We reduce F_n modulo m taking the least nonnegative residues, and let $k(m)$ denote the length of the period of the repeating sequence $F_n^*(m)$.

Defines the Fibonacci finite words as the contatenation of the two previous terms $f_n = f_{n-1}f_{n-2}$, $n > 1$, with initial values $f_0 = 1$ and $f_1 = 0$ and the infinite Fibonacci word f, $f = \lim f_n$.

Let $w_n^*(m)$ be the last $F_{n+1}^*(m)$ symbols of the word f_n. If $F_{n+1}^*(m) = 0$ for some n, then $w_n^*(m)$ is empty word. Let $f_0^*(m) = 1$ and for arbitrary integer n, $n \geq 1$, $f_n^*(m) = f_{n-1}^*(m)w_n^*(m)$. Denote by $w^*(m)$ the limit $f^*(m) = \lim_{n\to\infty} f_n^*(m)$. We say that $f_n^*(m)$ is a finite Fibonacci like periodic word (FLP-word) by modulo m and $f^*(m)$ is a infinite FLP-word by modulo m.

Theorem. *The infinite FLP-word $f^*(m)$ is a periodic word and sequence subwords $w_n^*(m)$ of $f^*(m)$ has period $k(m)$.*

An example of a non-Borel locally-connected finite-dimensional topological group

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Answering a question posed by S.Maillot in MathOverFlow, for every $n \in \mathbb{N}$ we construct a locally connected subgroup $G \subset \mathbb{R}^{n+1}$ of dimension n, which is not locally compact.

Paper reference: **http://arxiv.org/abs/1604.00149**

Translations of sets that belong to the ideal of meager null sets

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We present some recent results related to the properties of the intersection ideal $\mathcal{M} \cap \mathcal{N}$ in the Cantor space.

This is a continuation of an earlier paper published in CMUC.

Drawing graphs on few lines and few planes

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We investigate the problem of drawing graphs in 2D and 3D such that their edges (or only their vertices) can be covered by few lines or planes. We insist on straight-line edges and crossing-free drawings. This problem has many connections to other challenging graph-drawing problems such as small-area or small-volume drawings, layered or track drawings, and drawing graphs with low visual complexity. While some facts about our problem are implicit in previous work, this is the first treatment of the problem in its full generality. Our contribution is as follows.

1. We show lower and upper bounds for the numbers of lines and planes needed for covering drawings of graphs in certain graph classes. In some cases our bounds are tight; in some cases we are able to determine exact values.
2. We relate our parameters to standard combinatorial characteristics of graphs (such as the chromatic number, treewidth, maximum degree, or arboricity) and to parameters that have been studied in graph drawing (such as the track number or the number of segments appearing in a drawing).
3. We pay special attention to planar graphs. For example, we show that there are planar graphs that can be drawn in 3-space on a lot fewer lines than in the plane.

Homogeneity properties of a σ-ideal related to a Borel measure

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Given a σ-finite atomless Borel measure μ on a perfect compact metrizable space X, we consider the σ-ideal I_f of Borel sets in X that can be covered by countably many compact sets of finite measure.

It turns out that if X is not in I_f, then the partial order of Borel subsets of X not in I_f, ordered by inclusion, satisfies a certain strong homogeneity condition which guarantees, in particular, homogeneity of the forcing associated with this order.

On the other hand, for some X and μ there is a Borel set $Y \subseteq X$ not in I_f such that for any Borel map $f : X \to Y$ there is a compact subset set C of Y with $\mu(C) < \infty$ but $f^{-1}(C) \notin I_f$. This shows that the σ-ideal I_f lacks some other homogeneity conditions, distinguished by J. Zapletal.

Hyperspaces of compact convex sets: some new results

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We will consider the iterated hyperspaces of (max-plus) compact convex sets in Euclidean spaces, the hyperspaces of max-plus convex polyhedra, and some hyperspaces of bodies of constant width. Some results will concern maps of hyperspaces.

Some open topological problems in analysis

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The shadow problem: What is the smallest number of pairwise disjoint balls centered at the sphere S^{n-1} such that the union of the balls intersects any straight line passing through the center of the sphere?

This problem was considered by G. Khudaiberganov and was mentioned in the lectures of the professor L. Aizenberg, during summer mathematical schools in Kaciveli (Crimea). In our talk we consider some related problems:

1. The centers of balls are free (not on a fixed sphere)
2. Ray convexity
3. Family of convex sets with non empty interior
4. Complex and hypercomplex spaces
5. Shadow for every points of ball
6. Tangent to sphere bundles of lines
7. Consider equal radii of balls
8. Change transformation group
9. Not exclude pairwise intersecting family.

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On midpoint-free subsets of some topological groups

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A subset of an abelian group is *midpoint-free* if it contains no three distinct elements a, b, c such that $a+b=2c$. We study midpoint-free sets in various classical topological groups.

For every infinite cardinal $\kappa \leq \mathfrak{c}$, we show that the real line can be partitioned into κ-many maximal midpoint-free sets.

Examples of closed maximal midpoint-free subsets are given for topological groups $\mathbb{R}$, $\mathbb{C}$, $S^1 = \mathbb{R}/\mathbb{Z}$, $S^1 \times S^1$.

Finally, among sets that are not regular, such as nonmeasurable sets, Bernstein sets, and Luzin sets, we study instances which are midpoint-free.

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