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Section:
History of Mathematics

Stefan Banach and Lviv Mathematics

Yaroslav Prytula and Taras Banakh

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The University and the Polytechnic School were the main centres for teaching and conducting systematic scientific research in mathematics in Lviv. Promotions of Doctors of Philosophy in Mathematics began at the University since the 70s of the nineteenth century. The head of the Department of Mathematics back then was Wawrzyniec (Laurentius) Żmurko (1824-1889), who studied at the Lviv and Vienna Universities and the Vienna Polytechnic School.

Józef Puzyna (1856-1919), a student of W. Żmurko, played an outstanding role in the development of mathematics at the University for more than 30 years.

K. Kuratowski wrote about J. Puzyna: "He was to a certain degree a forerunner of ideas that would flourish in the writings of the next generation of Polish mathematicians". Weierstrass and Kronecker had a decisive influence on the subject of the studies of J. Puzyna who attended their lectures at the University of Berlin. The main work of his life was a two-volume monograph "Teoria funkcji analitycznych". Puzyna taught more than 30 different courses at the Lviv University, organized and supervised two seminars in 1894. Participants of those seminars subsequently became famous scientists, among them Volodymyr Levytskyi (1872-1956), Antoni Łomnicki (1881-1941), Otto Nikodym (1887-1974), Miron Zarycki (1889-1961), Stanisław Ruziewicz (1889-1941) and others. Polish scientific terminology, in particular, set theory terminology, was formed in the monograph of J. Puzyna, and the first scientific work in mathematics in the Ukrainian language was published by his student V. Levytskyi.

With J. Puzyna's support, a second mathematical department was opened at the University, which was headed by Jan Rajewski (1857-1906) in 1900-1906.

In 1908, J. Puzyna invited a graduate from Warsaw University, a student of Georgii Voronoi (1868-1908), Wacław Sierpiński, to Lviv. During the years 1908-1914 and 1918, W. Sierpiński taught some modern courses, in particular in the set theory, the theory of functions of a real variable, the measure theory and Lebesgue integration, the analytic number theory, and others.

Zygmunt Janiszewski (1888-1920) came to the University of Lviv on the advice of W. Sierpinski in 1913. In the same year, under the supervision of W. Sierpiński, Stefan Mazurkiewicz (1888-1945) and Stanisław Ruziewicz received a doctorate in Lviv. During the First World War, W. Sierpinski was interned in Russia and Z. Janiszewski served in Polish legions.

In 1917, J. Puzyna invited his former student Hugo Steinhaus (1887-1972) to habilitation at the University. Later, H. Steinhaus studied at Gottingen, where he received a doctorate under the supervision of David Hilbert. Stefan Banach (1892-1945) arrived in Lviv later on with H. Steinhaus.

Stefan Banach was born on 30 March 1892 in Kraków. Banach's parents were Stefan Greczek and Katarzyna Banach. At that time his father was a soldier in the Austrian Army, and his mother was a maid. K. Banach, being single and having no means of living, could not take care of her son. The boy was initially sent to his grandmother who lived in the village, and after a few months, his mother arranged for her son to be raised by Franciszka Płowa and her niece Maria, who lived in Kraków. F. Płowa owned a small laundry. S. Greczek remembered his son and kept in touch with the caretaker's family. Banach graduated from the gymnasium in Kraków in 1910 and enrolled for studies at the Polytechnic School in Lviv. At first, he was a student of the Faculty of Machine Construction, and from the second year – the Faculty of Engineering. He attended the lectures in mathematics by Zdzisław Krygowski (1872-1955), in general mechanics – by Alfred Denizot (1873-1937), in descriptive geometry – by Kazimierz Bartel (1882-1941), in technical mechanics - by Maksymilian Huber (1872-1950), and others. Being a student until 1914, Banach passed only the first state exam – 'half-exam', which included mathematics, descriptive geometry, physics, general and technical mechanics with elements of graphic statics.

It is possible that, while in Lviv, Banach was acquainted with the ideas that originated at that time among the university mathematicians. To a large extent, Banach acquired mathematical knowledge on his own, as well as means of living, teaching others.

Since the beginning of the First World War, Lviv was occupied by Russian troops and Stefan Banach returned to Kraków. He met H. Steinhaus in 1916 there. According to H. Steinhaus, while he was strolling through Planty gardens, he was surprised to overhear the term "Lebesgue measure" and walked over to investigate. There were two young people sitting on a bench. Those were S. Banach and O. Nikodym. Since then, close cooperation has begun between the two prominent personalities, Banach and Steinhaus. Banach's first scientific work, joint with Steinhaus, was published in 1918 and concerned the convergence of trigonometric series. The topics of Banach's first papers were trigonometric series, functional equations, and the theory of functions of a real variable.

Banach arrived in Lviv for a Steinhaus habilitation lecture at the University in 1917. In 1920, A. Łomnicki, who at that time served as the head of the II Department of Mathematics in Lviv Polytechnic, by Steinhaus' recommendation, suggested Banach to be an assistant of the Department. The Senate appointed Banach, on condition that he would have a doctor's diploma in six months.

Banach filed a doctoral thesis at the Academic Senate of the Faculty of Philosophy of Lviv University already in June 1920. The evaluation of thesis was written by Eustachy Żyliński (1889-1954) and H. Steinhaus. After passing exams in mathematics and physics (November 1) and philosophy (December 11) with honours, on January 22, 1921, an official "promotion" took place – granting the degree of Doctor of Philosophy. The promoter was Professor Kazimierz Twardowski (1866-1938).

The doctoral thesis of S. Banach became a turning point for a new mathematical discipline "functional analysis". It defined the "functional space", which allowed to combine various fields of mathematics: classical analysis, calculus of variations, differential equations, etc. Banach defined "B-type space" as a complete normed vector space, which is now called a Banach space. He showed with many examples that this concept covers all functional spaces known at that time, and began to construct a general theory of these spaces, in particular, proved the fixed-point theorem, which got many applications. Stanislaw Ulam (1909-1984) noted that those results covered more general spaces than those in the works of such mathematicians as Hilbert, E. Schmit, fon Neuman, F. Riss, et al. The doctoral thesis, published in 1922, became a powerful stimulus for the development of functional analysis.

After the publication of doctoral thesis, Banach got a new interest – the measure theory. He showed that, unlike the three-dimensional space, for the spaces of dimensions 1 and 2, the general measure problem has a solution. Banach-Tarski paradox, which was published later, became even more famous.

To obtain the license to lecture at the University, Banach was habilitated in the spring of 1922: he submitted a scientific paper in measure theory, passed a colloquium and delivered a lecture "Development of the concept of measure" for the Academic Senate of the Faculty. After approval of the habilitation, Banach received the position of Associate Professor (profesor nadzwyczajny) at the University in July 1922. He became a Professor (profesor zwyczajny) in 1927. There were four mathematical chairs in the University, headed by professors: E. Żyliński, H. Steinhaus, S. Ruziewicz, S. Banach. In 1924 Banach became a correspondence member of the Polish Academy of Sciences. He also received a scholarship for a one-year trip to France. The 1st Congress of Polish Mathematicians took place in Lviv in 1927; there were also foreign participants. After the congress, Banach and Steinhaus founded the journal

"Studia Mathematica" whose first volume was published in 1929. Nine volumes of the journal were published in Lviv, the last one in 1940.

The monograph "Théorie des opérations linéaires" published in 1932 had a great influence on the development of research in functional analysis in the world. It was published in Polish in 1931, and the Ukrainian translation by Miron Zarycki was released in 1948. The monograph quickly gained widespread recognition, since then Banach's approaches and terminology were accepted everywhere. During those years, a group of actively working mathematicians – mainly students of S. Banach and H. Steinhaus – was formed in Lviv. During 1920-1939, the Academic Senate of the Faculty granted a PhD to 13 mathematicians and 2 logicians. They were: 1921 – Stefan Banach, 1924 – Juliusz Pawel Schauder (1899-1943), Stefan Kaczmarz (1895-1939), Władysław Hikliborc (1899-1948), 1927 – Sala Weinlös (1903-??), 1928 – Władysław Orlicz (1903-1990), 1929 – Zygmunt Birnbaum (1903-2000), 1930 – Miron Zarycki (1889-1961), Herman Auerbach (1901-1942), 1932 – Stanisław Mazur (1905-1981), 1934 – Józef Schreier (1909-1943), 1937 – Marc Kac (1914-1984), Władysław Hetper (1909-1940), 1938 – Meier Eidelheit (1910-1943), Józef Pepis (1910-1943?). Stanisław Ulam (1909-1984) received a doctorate in mathematics in Lviv Polytechnic in 1933.

The favourite place for meetings of Lviv mathematicians was "Scottish Café". The collection of mathematical problems "Scottish Book" is connected with this place. S. Banach posed the first problem in this book on July 17, 1935. H. Steinhaus made the last record in the book on May 31, 1941.

Banach's invitation to the International Congress of Mathematicians in Oslo to deliver a plenary talk in 1936 was the expression of world recognition. The names of the mathematicians who visited Lviv, including Henri Lebesgue, John von Neumann, Ernst Zermelo, and others, testify to the importance of the city in the mathematical world.

The main achievements of the Lviv Mathematical School are: creating the fundamenta of functional analysis and developing its particular sections; introducing topological methods in functional analysis and their application to the problems of differential equations in partial derivatives; considering probability as a measure; introducing the concept of independent functions; solving the fundamental problems of measure theory; pioneering works in the game theory, topological algebra, computability theory.

The broad use of non-constructive methods based on Zermelo's axiom, Baire category and the Lebesgue measure was an attribute of the research for Lviv mathematicians. Their creative enthusiasm could be developed due to the specific atmosphere of collective work of teachers and students at seminars and in "Scottish Café".

S. Banach was appointed the Dean of the Faculty of Physics and Mathematics and the Head of the Department of Analysis I in the University, reorganized by the Soviet government in 1939. In 1939-1941, he had scientific trips to Kyiv, Moscow, Tbilisi, and Odesa. In March 1941, he was awarded a PhD in Physics and Mathematics and the title of professor by the Higher Certification Commission of the USSR.

During the German occupation, Banach, along with his son and many Lviv scientists, worked as a donor for the antitumor vaccine at the bacteriological institute of Professor P. Weigl. This work helped him with means of living.

After the restoration of the university in August 1944, Banach remained the head of the department but quitted being the dean of the faculty. He restored scientific contacts with Soviet and Polish mathematicians.

Since the spring of 1945, his disease became a critical issue. Banach died on August 31, 1945, was buried on September 3 at the Lychakiv cemetery in the crypt of the Riedl family, whose house he lived in since August 1944.

On Stefan Banach's visit to Georgia and two problems posed by him

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In [1] I wrote: "*At the beginning of the talk it will be commented an information about being of Stefan Banach in Tbilisi (June, 1941).*" Soon after submission of [1] in "Topology Atlas" professor Lech Maligranda wrote me in July 28, 2012: "*I know that Banach was in Tbilisi and Gori in 1941. My information is that it was in March but you are writing in June. How do you know this? Maybe you even know exact days of stay of Banach in Tbilisi and Gori?*"

The paper [2] contained some answers to Maligranda's questions. Namely, it is established that

1. On Saturday, March 15, 1941 "the delegation of Lvov State University, lead by S.S. Banach, the famous mathematician, arrived in Tbilisi ... Banach, the head of delegation said ... Our delegation will stay in Georgia for five days ..."
2. "Gori, March 19. The professors of Lvov University comrades Banach, Zarits'kyi, docent Braginets and lady-student Solyak arrived to Gori."

In the first part of my talk I'll give some further comments about S. Banach's visit to Georgia. The second part of the talk will deal with two problems posed by him (before his visit to Georgia). One of the problems is taken from W. Orlicz's (1903–1990) paper [3, p. 124], while the another one is [4, Problem 106]. I'll follow mainly [5] with some supplements related to the contribution of my colleagues from Georgia.

1. V. Tarieladze, *Banach-Saks property and permutations,* International Conference dedicated to the 120th anniversary of Stefan Banach. Abstracts of Reports, Lviv, Ukraine, 17-21 September, 2012, pp. 31–32.
2. V. Tarieladze, *Stefan Banach in Tbilisi (On the occasion of his 120-th Birthday Anniversary)*, III International Conference of the Georgian Mathematical Union. Book of Abstracts. Batumi, Georgia, 2-9 September, pp. 21–26.
3. W. Orlicz, Uber die unabhangig von der Anordnung fast uberall konvergenten Funktionenreihen, Bull. Int. Polon. Sci. Ser. A (1927), 117–125.
4. R. D. Mauldin (Ed.), *The Scottish Book, Mathematics from The Scottish Cafe*, Birkhauser, 1981.
5. M. Kadets and V. Kadets, *Series in Banach spaces: conditional and unconditional convergence*, **94**, Birkhauser, 2012.

Section: Banach and Locally Convex Spaces

On the absolute convergence of the series of Fourier coefficients

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We study the absolute convergence of the series of Fourier coefficients with respect to Haar-like systems for functions of the class generalized bounded variation $BV(p(n) \uparrow p, \varphi)$. Denote by $a_m(f)$ the Fourier cofficients of the function $f \in L[0,1]$ with respect to Haar-like systems, i.e. $a_m(f) = \int_0^1 f(t)\bar{\chi}_m(t)dt$, $(m \in \mathbb{N})$. We study the question of convergence of the series $\sum_{m=1}^{\infty} m^{\alpha}|a_m(f)|^{\beta}$ in the class $BV(p(n) \uparrow p, \varphi)$.

Symmetric analytic functions of unbounded type on Banach spaces

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It is known that there are entire functions on infinite dimensional Banach spaces which are not bounded on some bounded subsets – the functions of unbounded type. Entire functions of unbounded type were investigated by many authors. For example, in [1] it was proved that there exists an entire function on a Banach space such that it is bounded on one of the two disjoint balls and unbounded on the other. However, the question about existence of symmetric entire functions of unbounded type on infinite dimensional Banach spaces so far has been open. We construct an example of a symmetric analytic function on ℓ_1 which is not of bounded type.

1. J. M. Ansemil, R. M. Aron, and S. Ponte, *Behavior of entire functions on balls in a Banach space*, Indag. Math. (N.S.) **20** (2009), 483–489.

Linear operators reversing orthogonality in normed spaces

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We consider linear operators $T\colon X \to X$ on a normed space X which reverse orthogonality, i.e., satisfy the condition

$$x \perp y \quad \Longrightarrow \quad Ty \perp Tx, \qquad x, y \in X,$$

where $\perp$ stands for Birkhoff orthogonality.

Paper reference:
http://aot-math.org/article_38478_c15ca13cf82bd7c234123b9bb787e61.pdf.

Application of wavelet basis expansions to description of some classes of smooth functions

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Let us consider an orthogonal trigonometric Schauder basis $\{t_k\}_{k=0}^{\infty}$ in the space $L^p_{2\pi}$ [1]. This basis was constructed by using ideas of a periodic multiresolution analysis and corresponding wavelet spaces. A function $f \in L^p_{2\pi}$ can be represented by a series

$$f = \sum_{k=0}^{\infty} \langle f, t_k \rangle t_k$$

converging in the norm of the space $L^p_{2\pi}$, where the coefficient functionals $\langle f, t_k \rangle$ are Fourier coefficients of f with respect to the basis $\{t_k\}_{k=0}^{\infty}$:

$$\langle f, t_k \rangle = \frac{1}{2\pi} \int_0^{2\pi} f(x)\overline{t_k(x)} \mathrm{d}x.$$

In [2] we obtained characterization of Besov spaces in terms of summability conditions imposed on the coefficients $\langle f, t_k \rangle$. Moreover, we showed that smooth properties of a function f from Besov spaces in the neighborhood of some point $x_0 \in [0, 2\pi)$ can be described by using only few coefficients $\langle f, t_k \rangle$ and presented a way of the selection of these coefficients.

1. J. Prestin and K. K. Selig, *On a constructive representation of an orthogonal trigonometric Schauder basis for $C_{2\pi}$*, Oper. Theory, Adv. Appl. **121** (2001), 402–425.
2. N. Derevianko, V. Myroniuk, and J. Prestin, *Characterization of local Besov spaces via wavelet basis expansions*, Front. Appl. Math. Stat. **3**:4 (2017), **doi: 10.3389/fams.2017.00004**.

Paper reference: (**https://doi.org/10.3389/fams.2017.00004**).

The coincidence of scalar and strong almost periodicity for uniformly continuous functions in Banach spaces not containing c_0

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In 1963 Lyubich proved that if a separable Banach space X is weakly sequentially complete, then every scalarly almost periodic one-parametric group of linear transformations of X is strongly almost periodic. In the case of the space $X = c$ the theorem is not true. A significant result of this problem was obtained by M.I. Kadets in Ukrainian Mathematical Journal, Vol. 49, No. 4, 1997. In the present work, the author continues the research mentioned in that paper and has obtained a new result as the following:

Let a function $f(t) : \mathbb{R} \to X$ be uniformly continuous and scalarly almost periodic. Then it is almost periodic if, either the space X does not contain subspaces isomorphic to the space c_0 or the set of values of the function is weakly relative complete.

Linearization of holomorphic semicocycles

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The talk is devoted to semicocycles over semigroups of holomorphic self-mappings. They appear naturally in the study of the assymptotic behavior of non-autonomous dynamical systems in Banach spaces.

Simplest semicocycles are those independent of the space-variable. So, the problem (called the linearization problem) is to establish whether a semicocycle is cohomologous to such independent one.

Focusing on the linearization problem, we provide some criteria for a semicocycle to be linearizable as well as several easily verifiable sufficient conditions. These conditions are essential even for semicocycles over linear semigroups.

When is the sum of complemented subspaces complemented?

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We provide a sufficient condition for the sum of a finite number of complemented subspaces of a Banach space X to be complemented. Under this condition a formula for a projection onto the sum is given. We also show that the condition is sharp (in a certain sense). Special attention is paid to the case when X is a Hilbert space; in this case we get more precise results.

Paper reference: (**http://arxiv.org/abs/1606.08048**).

A remark on the sum of a narrow and a finite rank orthogonally additive operators

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One of our main results asserts that, the sum of a narrow and a finite rank laterally-to-norm continuous orthogonally additive operators acting from a Köthe Banach space with an absolutely continuous norm on a finite atomless measure space to a normed space, is narrow.

Narrow operators were first systematically studied in [5], and orthogonally additive operators for the setting of Riesz spaces (= vector lattices) in [2] and [3]. Narrow orthogonally additive operators were introduced and studied in [6]. Let E be a Riesz space and F a real linear space. A function $T : E \to F$ is called an *orthogonally additive operator* if $T(x+y) = T(x)+T(y)$ whenever $x, y \in E$ are *orthogonal*, that is, $|x| \wedge |y| = 0$ (write $x \perp y$). The notation $z = x \sqcup y$ means that $z = x + y$ and $x \perp y$. An element x of a Riesz space E is called a *fragment* of an element y provided $x \perp (y - x)$ (write $x \sqsubseteq y$). Observe that if $z = x \sqcup y$ then x and y are orthogonal fragments of z. Let E be an atomless Riesz space and X a normed space. A function $f : E \to X$ is called *narrow at a point* $e \in E$ if for every $\varepsilon > 0$ there exists a decomposition $e = e_1 \sqcup e_2$ of e such that $\|f(e_1) - f(e_2)\| < \varepsilon$; f is called *narrow* if it is narrow at every point $e \in E$.

The question of whether the sum of a narrow and a compact narrow operator is narrow, is substantial even for linear operators [7, Problem 5.6]. This problem was solved in the positive for wide classes of the domain and range spaces [4]. In most natural cases (e.g., if the the domain lattice is a Köthe F-space with an absolutely continuous norm on the unit on a finite atomless measure space then every compact linear operator is narrow [7, Proposition 2.1], however in general this is not true: there is a nonnarrow continuous linear functional on L_∞ [7, Example 10.12]. We study similar questions for a more general setting of orthogonally additive operators. The results presented below refine the main result of [1].

Recall that, a family $(r_i)_{i\in I}$ in a Boolean algebra $\mathcal{B}$ is said to be *independent* if $\bigcap_{j\in J} \theta_j r_j \neq \mathbf{0}$ for every finite subset $J \subset I$ and every collection of signs $\theta_j = \pm 1$, $j \in J$. We say that, a family $(r_i)_{i\in I}$ in a Boolean algebra $\mathcal{B}$ is *vanishing at infinity* if $\bigcap_{j\in J} \theta_j r_j = \mathbf{0}$ for every infinite subset $J \subseteq I$ and every collection of signs $\theta_j = \pm 1$, $j \in J$. We say that, a Boolean algebra $\mathcal{B}$ is *countably divisible* if for every $0 < b \in \mathcal{B}$ the boolean algebra $\mathcal{B}_b = \{a \in \mathcal{B} : a \le b\}$ contains an independent vanishing at infinity sequence. We show that, for instance, every atomless measurable Boolean algebra is countably divisible. We say that, a Riesz space is *countably divisible* if for every $e \in E^+$ the Boolean algebra $\mathfrak{F}_e$ of all fragments of e is countable divisible. As a consequence of the above result, one obtains that every Köthe Banach space with an absolutely continuous norm on a finite atomless measure space is a countably divisible Riesz space.

The following theorem is our main result.

Theorem. *Let E be an atomless Dedekind complete countably divisible Riesz space, X a normed space, $S,T : E \to X$ orthogonally additive operators. If S is narrow and T is finite rank laterally-to-norm continuous the the sum $S+T$ is narrow.*

Corollary. *Let E be a Köthe Banach space with an absolutely continuous norm on a finite atomless measure space, X a normed space, $S,T : E \to X$ orthogonally additive operators. If S is narrow and T is finite rank laterally-to-norm continuous then the sum $S+T$ is narrow.*

1. H. I. Gumenchuk, *On the sum of narrow and finite rank orthogonally additive operators*, Ukr. Math. J. **67**:12 (2015), 1831–1837.
2. J. M. Mazón and S. Segura de León, *Order bounded orthogonally additive operators*, Rev. Roumane Math. Pures Appl. **35**:4 (1990), 329–353.
3. J. M. Mazón and S. Segura de León, *Urysohn operators*, Rev. Roumane Math. Pures Appl. **35**:5 (1990), 431–449.
4. V. Mykhaylyuk, *On the sum of a compact and a narrow operators*, J. Funct. Anal. **266** (2014), 5912–5920.
5. A. M. Plichko and M. M. Popov, *Symmetric function spaces on atomless probability spaces*, Diss. Math. (Rozpr. mat.) **306** (1990), 1–85.
6. M. A. Pliev and M. M. Popov, *Narrow orthogonally additive operators*, Positivity, **18** (2014), 641–667.
7. M. Popov and B. Randrianantoanina, *Narrow operators on function spaces and vector lattices*, Berlin-Boston, De Gruyter, 2013.

Spectra of algebras of entire functions, generated by the sequence of polynomials on a Banach space

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Let X be a complex Banach space. Let $\mathbb{P} = \{P_1, \ldots, P_n, \ldots\}$ be such that P_n is an n-homogeneous continuous complex-valued polynomial on X for every positive integer n and the elements of $\mathbb{P}$ are algebraically independent. Let us denote $H_{\mathbb{P}}(X)$ the closed subalgebra, generated by the elements of $\mathbb{P}$, of the the Fréchet algebra $H_b(X)$ of all entire functions of bounded type on X. Note that every $f \in H_{\mathbb{P}}(X)$ can be uniquely represented in the form

$$f(x) = f(0) + \sum_{n=1}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} a_{k_1\ldots k_n} P_1^{k_1}(x) \cdots P_n^{k_n}(x).$$

Consequently, every continuous homomorphism $\varphi : H_{\mathbb{P}}(X) \to \mathbb{C}$ is uniquely determined by its values on the elements of $\mathbb{P}$. Therefore, the spectrum of $H_{\mathbb{P}}(X)$ can be identified with some set of sequences of complex numbers.

In this work we consider the case, when X is a closed subspace of ℓ_∞, and describe spectra of some algebras $H_{\mathbb{P}}(X)$.

Symmetric fractional maps on Banach spaces

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A function f on complex ℓ_1 to $\mathbb{C}$ is said to be *symmetric* if for every permutation σ on positive integers $\mathbb{N}$, $f(\sigma(x)) = f(x_{\sigma(1)}, \ldots, x_{\sigma(n)}, \ldots) = f(x), x \in \ell_1$.

The main result is a construction of a fractional map Φ from ℓ_1 to itself which is symmetric and the composition operator $f \mapsto f \circ \Phi$ is a continuous homomorphism of the algebra of bounded type symmetric entire functions on ℓ_1 to the algebra of symmetric analytic functions on the unit ball of ℓ_1.

Differentiability of semicocycles in Banach spaces

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Semicocycles play important role in the theory of dynamical systems and are closely connected to semigroups of weighed composition operators. We consider semicocycles defined on the open unit ball in a complex Banach space and taking values in a Banach algebra.

We study semicocycle properties employing, in particular, their link with semigroups. On the other hand, we discover a dissimilarity of these two classes.

The main question we address in the talk is the differentiability of semicocycles provided by different kinds of the continuity assumptions.

Read's solution of finite-codimensional proximinal subspace problem: properties, simplifications and extensions

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A subset Y of a Banach space X is said to be *proximinal*, if for every $x \in X$ there is a $y \in Y$ such that $\|x - y\| = \operatorname{dist}(x, Y)$. Recently, Charles J. Read in his paper "Banach spaces with no proximinal subspaces of codimension 2" (to appear in Israel Journal of Mathematics) constructed an equivalent norm $|||\cdot|||$ on c_0 such that the space $\mathcal{R} = (c_0, |||\cdot|||)$ does not have proximinal subspaces of codimension 2, thus solving an open since 1974 problem by Ivan Zinger.

In Theorem 4.2 of his paper "Norm-attaining functionals need not contain 2-dimensional subspaces" [Journal of Functional Analysis **272** (2017), 918–928] Martin Rmoutil demonstrated that the same space $\mathcal{R}$ gives the negative solution to the following problem by Gilles Godefroy (2001): is it true that for every infinite-dimensional Banach space X the set $NA(X) \subset X^*$ of norm-attaining functionals contains a two-dimensional linear subspace?

In this talk I am going to recall the classical results in this field, to present the Read's construction, to explain the relationship between Read's and Rmoutil's results, to present our joint with Ginés López and Miguel Martín paper "Some geometric properties of Read's space" (to appear in Journal of Functional Analysis), and to speak about our current project with Ginés López, Miguel Martín and Dirk Werner about simplification and extension of Read's construction.

Towards theory of frames in Krein spaces

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During the first part of talk we discuss the difference between two contemporary definitions of frames in Krein spaces. The first definition, was introduced in 2012 by Julian I. Giribet et al. [1] and called J-frame. The second one was defined in 2015 by Kevin Esmeral et al. [2]. We briefly mention conclusions related to these definitions and properties of associated frames operators. In the second part, according to [3], we introduce a new definition of frames in Krein spaces which generalizes the previous ones and fits well (in our opinion) the ideology of Krein spaces.

1. J. I. Giribet, A. Maestripieri, F. Martinez Peria, and P. G. Massey, *On frames for Krein spaces*, J. Math. Anal. Appl. **393** (2012).
2. K. Esmeral, O. Ferrer, and E. Wagner, *Frames in Krein spaces arising from a non-regular W-metric*, Banach J. Math. Anal. **9**:1 (2015) 1–16.
3. S. Kużel and A. Kamuda, Towards theory of frames in Krein spaces, (in preparation).

Equivalent definitions of Sobolev spaces on LCA groups

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We investigate Sobolev spaces on locally compact abelian groups (LCA groups). In our current studies we consider two definitions of Sobolev spaces. The first one is formulated via the Fourier transform on a LCA group. The second one requires a notion of differential operator on a LCA group, introduced by A. Wawrzyńczyk in Colloquium Mathematicum Vol. 19 (1968). We show that both definitions are equivalent under some natural conditions.

Order intervals in Banach lattices, a condition of Lozanowskij, and a Lyapunov type convexity theorem

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The set of extreme points of an order interval in a Banach lattice X is strongly closed. If X is atomic, it is even weakly closed. The converse holds provided X has order continuous norm. On the other hand, if X satisfies a condition of G. Ja. Lozanovskij (1978), which is a topological version of nonatomicity, then the set of extreme points of an order interval is weakly dense in that interval. The proof involves a Lyapunov type convexity theorem. If X has order continuous norm, then X is nonatomic if and only if it satisfies the Lozanowskij condition if and only if the set of extreme points of every order interval in X is weakly dense in that interval.

On polynomial ω-ultradistributions

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We construct some polynomial analogues of ω-ultradistributions [1]. We investigate a generalized operation of differentiation, a group of shifts and a convolution in the space of polynomial ω-ultradistributions.

1. R. W. Braun, R. Meise, and B. A. Taylor, *Ultradifferentiable functions and Fourier analysis*, Results in Math. **17** (1990), 206–237.

Stability of Schauder decompositions

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The concept of Schauder decomposition is a natural generalization of the concept of Schauder basis in a Banach space. Schauder decompositions are effective tools of modern functional analysis and infinite-dimensional linear systems theory. It is caused by the fact that every such decomposition, if it exists, provides the decomposition of some Banach space X into an infinite direct sum of closed subspaces of X and it may be very convenient to operate with these subspaces separately.

We obtain a number of stability theorems in terms of closeness of projections for Schauder decompositions in Banach spaces with certain geometrical properties. More precisely, such spaces should contain the so-called Schauder-Orlicz decomposition, which can be treated as a Schauder decomposition possessing the generalised property of orthogonality. These studies develop and extend the result of T. Kato on similarity for sequences of projections in a Hilbert space (Bull. Amer. Math. Soc., 1967) to the case of Banach spaces and Riesz bases of subspaces in Hilbert spaces.

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Hypercyclic translation operator on spaces of analytic functions

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Let X be a Banach space and $H_b(X)$ be a space of analytic functions which are bounded on bounded subspaces. We consider conditions of hypercyclicity of translation operator $T_a : H_b(X) \to H_b(X)$, $T_a \colon f(x) \mapsto f(x+a), \quad x, a \in X, \quad a \neq 0$. If $H_b(X)$ is not separable space then T_a is not hypercyclic. In [1] it was obtained sufficient conditions for a bilateral weighted shift to be hypercyclic in the weak topology on ℓ_p. We are interested in a "weak" version of hypercyclicity of T_a.

1. K. C. Chan and R. Sanders, *A weakly hypercyclic operator that is not norm hypercyclic*, J. Operator Theory **52** (2004), 39–59.

Topological problems connected with approximation of continuous functions on Banach spaces

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Separating polynomials were first introduced by J. Kurzweil [2] in 1954 for approximating continuous functions on real separable Banach spaces. For approximation of uniformly continuous functions on a wider subclass of real Banach spaces than for continuous functions in the work [1], uniformly analytic separating functions were introduced. The case of approximation on a complex Banach space is dissipated in the work [3]. In [4] a connection between the existence of separating polynomials on a Banach space and a weak polynomial topology considered. During the talk we discuss topological problems associated with the approximation of continuous functions in Banach spaces. Also, we consider some special questions [3].

1. M. C. Boiso and P. Hájek, *Analytic approximations of uniformly continuous functions in real Banach spaces*, J. Math. Anal. Appl. **256** (2001), 80–98.
2. J. Kurzweil, *On approximation in real Banach spaces*, Studia Math. **14** (1954), 214–231.

3. M. A. Mitrofanov, *Approximation of continuous functions on complex Banach spaces*, Math. Notes. **86**:4 (2009), 530–541.
4. A. V. Zagorodnyuk and M. A. Mytrofanov, *Weak polynomial topology and weak analytic topology on Banach spaces and Fréchet spaces*, Carpathian Math. Publ. **4**:1 (2012), 49–57.

Distance of convergence in measure on L^0

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In the classical setting of a measure space, we introduce how the well-known topology of convergence in measure on L^0 can be quantified by a canonical approach structure which we call the distance of convergence in measure on L^0.

Quantitative versions of the Bishop-Phelps-Bollobás theorem

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In this talk we relate the Bishop-Phelps-Bollobas modulus with the modulus of uniform non-squareness. We demonstrate that the spherical Bishop-Phelps-Bollobas modulus $\Phi_X(\varepsilon)$ of a Banach spaces X can be estimated from above through the parameter of uniform non-squareness $\alpha(X)$ of X as follows: $\Phi_X(\varepsilon) \leq \sqrt{2\varepsilon}\,\sqrt{1-\frac{1}{3}\alpha(X)}$ for small ε.

Also we consider an analogue of the Bishop-Phelps-Bollobas moduli for the vector-valued case. D. Acosta, Richard M. Aron, Domingo Garcia, and Manuel Maestre proved that if Y has property β with parameter ρ, then for any Banach space X the pair (X, Y) has the Bishop-Phelps-Bollobas property for operators. We show that under these conditions the Bishop-Phelps-Bollobas modulus can be estimated as follows: $\Phi_{(X,Y)}(\varepsilon) \leq \sqrt{2\varepsilon}\sqrt{\frac{1+\rho}{1-\rho}}$.

The algebra of symmetric $*$-polynomials on $\mathbb{C}^2$

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Let X be a complex Banach space. A mapping $A : X^{p+q} \to \mathbb{C}$, where p and q are non-negative integers, is called a (p,q)-linear form if A is linear with respect to first p arguments and antilinear with respect to last q arguments.

A mapping $P : X \to \mathbb{C}$ is called a (p,q)-polynomial if there exist non-negative integers p and q and a (p,q)-linear mapping A_P such that

$$P(x) = A_P(\underbrace{x, \ldots, x}_{p+q})$$

for every $x \in X$.

A mapping $P : X \to \mathbb{C}$ is called a $*$-polynomial if it can be represented in the form

$$P = \sum_{p=0}^{K} \sum_{q=0}^{M} P_{p,q},$$

where K and M are non-negative integers and $P_{p,q}$ are (p,q)-polynomials.

We restrict our attention to the case $X = \mathbb{C}^2$. A $*$-polynomial $P : \mathbb{C}^2 \to \mathbb{C}$ is called symmetric if $P((z_1, z_2)) = P((z_2, z_1))$ for every complex numbers z_1 and z_2.

We investigate the algebra of all symmetric $*$-polynomials on $\mathbb{C}^2$.

Topology on the spectrum of the algebra of entire symmetric functions of bounded type on the complex L_∞

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Let L_∞ be the complex Banach space of all Lebesgue measurable essentially bounded complex-valued functions x on $[0,1]$ with norm

$$\|x\|_\infty = \operatorname{ess\,sup}_{t \in [0,1]} |x(t)|.$$

Let Ξ be the set of all measurable bijections of $[0,1]$ that preserve the measure. A function $f : L_\infty \to \mathbb{C}$ is called symmetric if for every $x \in L_\infty$ and for every $\sigma \in \Xi$ $f(x \circ \sigma) = f(x)$.

Let $H_{bs}(L_\infty)$ be the Fréchet algebra of all entire symmetric functions $f : L_\infty \to \mathbb{C}$ which are bounded on bounded sets endowed with the topology of uniform convergence on bounded sets.

Theorem 1. *Polynomials $R_n : L_\infty \to \mathbb{C}$, $R_n(x) = \int_{[0,1]}(x(t))^n\,dt$ for $n \in \mathbb{N}$, form an algebraic basis in the algebra of all symmetric continuous polynomials on L_∞.*

Since every $f \in H_{bs}(L_\infty)$ can be described by its Taylor series of continuous symmetric homogeneous polynomials, it follows that f can be uniquely represented as

$$f(x) = f(0) + \sum_{n=1}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1,\ldots,k_n} R_1^{k_1}(x) \cdots R_n^{k_n}(x).$$

Consequently, for every non-trivial continuous homomorphism $\varphi : H_{bs} \to \mathbb{C}$, taking into account $\varphi(1) = 1$, we have

$$\varphi(f) = f(0) + \sum_{n=1}^{\infty} \sum_{k_1+2k_2+\ldots+nk_n=n} \alpha_{k_1,\ldots,k_n} \varphi(R_1)^{k_1} \cdots \varphi(R_n)^{k_n}.$$

Therefore, φ is completely determined by the sequence of its values on R_n : $(\varphi(R_1), \varphi(R_2), \ldots)$.

By the continuity of φ, the sequence $\{\sqrt[n]{|\varphi(R_n)|}\}_{n=1}^{\infty}$ is bounded. On the other hand, we have

Theorem 2. *For every sequence $\xi = \{\xi_n\}_{n=1}^{\infty} \subset \mathbb{C}$ such that $\sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|} < +\infty$, there exists $x_\xi \in L_\infty$ such that $R_n(x_\xi) = \xi_n$ for every $n \in \mathbb{N}$ and $\|x_\xi\|_\infty \le \frac{2}{M} \sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|}$, where $M = \prod_{n=1}^{\infty} \cos\left(\frac{\pi}{2n+2}\right)$.*

Hence, for every sequence $\xi = \{\xi_n\}_{n=1}^{\infty}$ such that $\sup_{n\in\mathbb{N}} \sqrt[n]{|\xi_n|} < +\infty$ there exists a point-evaluation functional $\varphi = \delta_{x_\xi}$ such that $\varphi(R_n) = \xi_n$ for every $n \in \mathbb{N}$. Since every such a functional is a continuous homomorphism, it follows that the spectrum (the set of all continuous complex-valued homomorphisms) of the algebra $H_{bs}(L_\infty)$, which we denote by M_{bs}, can be identified with the set of all sequences $\xi = \{\xi_n\}_{n=1}^{\infty} \subset \mathbb{C}$ such that $\{\sqrt[n]{|\xi_n|}\}_{n=1}^{\infty}$ is bounded.

There are different approaches to the topologization of the spectra of algebras. The most common approach is to endow the spectrum by the so-called Gelfand topology (the weakest topology, in which all the functions $\widehat{f} : M_{bs} \to \mathbb{C}$, $\widehat{f}(\varphi) = \varphi(f)$, where $f \in H_{bs}(L_\infty)$, are continuous). Let us consider another natural topology on M_{bs}. Let $\nu : L_\infty \to M_{bs}$ be defined by $\nu(x) = (R_1(x), R_2(x), \ldots)$.

Let τ_∞ be the topology on L_∞, generated by $\|\cdot\|_\infty$. Let us define an equivalence relation on L_∞ by $x \sim y \Leftrightarrow \nu(x) = \nu(y)$. Let τ be the quotient topology on $M_{bs} : \tau = \{\nu(V) : V \in \tau_\infty\}$.

Note that ν is a continuous open mapping. Therefore, τ contains the Gelfand topology.

Theorem 3. $(M_{bs}, +, \tau)$ *is an abelian topological group, where "+" is the operation of coordinate-wise addition.*

Problems related to symmetric analytic functions on Banach spaces

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We consider algebras of analytic functions on complex Banach spaces that are invariant under the action of a certain set of operators acting on the given space. Such invariant functions have been vaguely called "symmetric". We compare the cases of Banach spaces like ℓ_p and L_p for various types of the groups of symmetry and discuss some common problems.

In particular, we are interesting to describe spectra of such algebras and analytic structures on the spectra.

Section: Operator Theory

Transformations of Nevanlinna operator-functions and their fixed points

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We consider certain classes of operator-valued Nevanlinna functions and give their characterization and realizations as compressed resolvents of selfadjoint operators. We consider two transformations of Nevanlinna functions and characterize their fixed points. In the scalar case we show connections of one of this transformation with Hamiltonian systems.

Construction and investigation of discrete Chebyshev polynomials

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Discrete Chebyshev polynomials (DCPs) are defined as polynomials of min-max deviation from zero over a discrete set of nodes on the real axis. They have the standard alternation property at the given points. Here we concentrate on two aspects.

First, we discuss the construction of DCPs of higher order (say, several hundred) via solving an ill-conditioned optimization problem. With this approach, using computer algebra with high numerical precision is mandatory but not sufficient, because the coefficients to be determined vary over an extremely large range of magnitudes. We show how to diagnose this difficulty and how to apply scaling to make the optimization work.

Numerical investigations show that the Vandermonde matrix associated with a DCP basis over integer nodes is extremely stable, i.e., its inverse appears to be uniformly bounded independent of its dimension. We discuss and illustrate some aspects of the proof of this fact (which by now has not been completed).

KB-operators and WKB-operators on Banach lattices

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Aqzzouz, Moussa and Hmichane proved that an operator T from a Banach lattice E into a Banach space X is b-weakly compact if and only if $\{Tx_n\}_n$ is norm convergent for every positive increasing sequence $\{x_n\}_n$ of the closed unit ball B_E of E. In the present paper, we introduce and study new classes of operators that we call KB-operators and WKB-operators. A continuous operator T from a Banach lattice E into a Banach space X is said to be KB-operator (respectively, WKB-operator) if $\{Tx_n\}_n$ has a norm (respectively, weak) convergent subsequence in X for every positive increasing sequence $\{x_n\}_n$ in the closed unit ball B_E of E. We investigate conditions under which KB-operators (respectively, WKB-operators) must be b-weakly compact. The collection of KB-operators and WKB-operators will be denoted by $L_{KB}(E,X)$ and $W_{KB}(E,X)$. The collection of b-weakly compact operators will be denoted by $W_b(E,X)$ and the collection of weakly compact and compact operators will be denoted by $W(E,X)$ and $K(E,X)$. Clearly $K(E,X) \subset W(E,X) \subset W_b(E,X) \subset L_{KB}(E,X) \subset W_{KB}(E,X)$. We will prove that if E is a KB-space, then $W_b(E,X) = L_{KB}(E,X)$ for each Banach space X.

In the following, we have some questions which until now remain open:

i) Give an operator T from a Banach lattice E into a Banach space X which is a KB-operator, but is not b-weakly compact.
ii) Give an operator T from a Banach lattice E into a Banach space X which is a WKB-operator, but is not a KB-operator.

The boundary value problem for the differential-operator equation of the order 2n with involution

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Recently, the number of publications using the involution operator in different sections of the theory of differential equations, integral equations, mathematical physics, inverse thermal conductivity problems for differential equations with fractional derivatives, the theory of self-similar functions, and pt-symmetric operators has increased significantly. The case of differential equations of high order with operators is less investigated [1].

In this talk, the problem with self-adjoint conditions for a differential-operator equations of order $2n$ with involution is studied. Eigenfunction expansion of the solution of the problem is found and it is proved that the system of eigenfunctions of the problem forms the Riesz basis.

Let $A : D(A) \subset H \to H$ be the operator with a point spectrum

$$\sigma_p(A) = \{z_k : z_k = a(k^\alpha),\ k \to \infty,\ a,\ \alpha > 0\},$$

and a system of eigenfunctions

$$V(A) = \{v_k \in H : Av_k = z_k v_k,\ k = 1, 2, \ldots\}$$

that forms the Riesz basis in the space H,

$$H(A^s) \equiv \{h \in H\colon A^s h \in H\},\ s \geq 0,$$

$$H_1 \equiv \{u(t)\colon (0,1) \to H,\ \|u(t)\|_H \in L_2(0,1)\},$$

$D_t\colon H_1 \to H_1$ is a strong derivative in the space H_1, in the sense

$$\lim_{\Delta t \to 0} \left\| \frac{u(t+\Delta t) - u(t)}{\Delta t} - D_t u(t); H_1 \right\| = 0,$$

$$H_2 \equiv \left\{\mathrm{u} \in \mathrm{H}_1,\ \mathrm{D}_\mathrm{t}^{2\mathrm{n}}\mathrm{u(t)} \in \mathrm{H}_1,\ \mathrm{A}^{2\mathrm{n}}\mathrm{u(t)} \in \mathrm{H}_1\right\},$$

I is the operator of involution in H_1, $Iu(t) \equiv u(1-t)$.

We consider the boundary problem

$$\begin{aligned} Lu \equiv (-1)^n D_t^{2n} u(t) + A^{2n} u(t) + \sum_{j=1}^{n} b_j \big(D_t^{2j-1} u(t) + \\ + D_t^{2j-1} u(1-t)\big) = f(t),\ t \in (0,1), \end{aligned} \tag{1}$$

$$\ell_j u \equiv \ D_t^{m_j} u(0) + \ \beta_j D_t^{m_j} u(1) = h_j, \ \beta_s \equiv (-1)^{m_s}, \ \beta_{n+s} \equiv (-1)^{m_{n+s}}, \quad (2)$$
$$b_s \in \mathbb{R}, \ j = 1, 2, \ldots, 2n, \ s = 1, 2, \ldots, n.$$

Theorem. *Let the boundary conditions* (2) *be self-adjoint and strongly regular in the sense of Birkhoff,* $h_j \in H\left(A^{k_j}\right), \ k_j = 2n - m_j - \frac{1}{2}, \ j = 1, 2, \ldots, 2n.$ *Then the problem* (1), (2) *has a unique solution and the system of eigenfunctions of the operator of the problem* (1), (2) *forms a Riesz basis in the space* H_1.

1. Ya. O. Baranetskij and A. A. Basha, *Nonlocal multipoint problem for differential-operator equations of order* $2n$, J. Math. Sci. **217**:2 (2016), 176–186.

Branching of resolvent

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Let $H = L^2_\rho(0, \infty)$, $\rho(\tau) > 0$. Suppose that interval $[0; \infty)$ coincides with continuous spectrum of some operator $T : H \to H$, $D(T) = H$, whose resolvent is denoted by $T_\zeta = (T - \zeta)^{-1}$. Bilinear form of the resolvent $(T_\zeta\varphi, \psi)$ is analytic function if $\zeta \notin [0; \infty)$. Suppose that there exists a linear space $\Phi \subset H$, $\bar{\Phi} = H$ such that the form $(T_\zeta\varphi, \psi)$, $\varphi, \psi \subset \Phi$, admits an analytic prolongation over the axis $(0; \infty)$. Denote $\Omega = \{\zeta : dist(\zeta, [0, \infty)) < \epsilon\}$, $\epsilon > 0$. Let $a(\zeta)$, $b(\zeta)$, $r(\zeta)$ be some functionals in H and $B(\zeta) : \Phi \to \Phi$ be some operator.

Definition 1. We say that an element $h_\zeta, \zeta \in \Omega/[0, \infty)$ *separates the branching* of the resolvent $T_\zeta, \zeta \in \Omega/[0, \infty)$ if

$$T_\zeta\varphi = (\varphi, b(\bar{\zeta}))h_\zeta + B(\zeta)\varphi, \ \varphi \in \Phi, \ \zeta \in \Omega/[0, \infty), \quad (1)$$

where the scalar functions $(\varphi, b(\bar{\zeta}))$ and the operator $B(\zeta)\varphi$, $\varphi \in \Phi$ are analytic in Ω and $h_\zeta \in H$ for $\zeta \in \Omega/[0, \infty)$ analytic on ζ element in H. We say that scalar function $M(\zeta)$, $\zeta \in \Omega/[0, \infty)$ *separates the branching* of h_ζ if $(h_\zeta, \psi) = M(\zeta)(a(\zeta), \psi) + (r(\zeta), \psi)$, $\psi \in \Phi$, $\zeta \in \Omega/[0, \infty)$, where functions $(a(\zeta), \psi)$ and $(r(\zeta), \psi)$ are analytic in Ω.

Lemma. *The functionals* $a(\sigma), (b(\sigma))$ *in* (1) *are eigenfunctionals of the operator* T (T^*), *corresponding to the point* $\sigma \in (0, \infty)$ *of continuous spectrum.*

Definition 2. An extension $T_{\max} \supset T$ is called *maximal operator* for T if:

1) 1) for every element $\varphi \in \Phi$ and every value $\sigma > 0$ there exists unique solution f_σ of the equation $(T_{\max} - \sigma)f_\sigma = \varphi$, $\sigma > 0$, $\varphi \in \Phi$ and $f_\sigma \in \Phi$.

We introduce the operator $T_{\max,\sigma} : \Phi \to \Phi$ as follows $D(T_{\max,\sigma}) = \Phi$, $T_{\max,\sigma}\varphi = (T_{\max} - \sigma)^{-1}\varphi = f_\sigma$, $\sigma > 0$;

2) The solution $T_{\max,\sigma}\varphi$ admits analytic prolongation f_ζ in the domain Ω such that $f_\zeta \in D(T_{\max})$ and $(T_{\max} - \zeta)f_\zeta = \varphi, \varphi \in \Phi$.
Denote $T_{\max,\zeta}\varphi = f_\zeta$, then

$$(T_{\max} - \zeta)T_{\max,\zeta}\varphi = \varphi, \quad \varphi \in \Phi, \quad \zeta \in \Omega,$$

where functions $(a(\zeta), \psi)$ and $(r(\zeta), \psi)$ are analytic in Ω.

Let us consider the differential expression

$$ly = -y'' + q(x)y, \; y(0) = 0.$$

We denote corresponding operator by $M = L + Q$, where $Ly = l(y)$, $Qy(x) = q(x)y(x)$, $x > 0$. Sin-Fourier transform gives the operator $T = S + V$.

The resolvent $M_\zeta = (M - \zeta)^{-1}$ is (see notations in [1]):

$$(M_\zeta z) = \frac{e(x, \sqrt{\zeta})}{e(\sqrt{\zeta})} \int\limits_0^x s(t, \zeta)z(t)dt + s(x, \zeta) \int_x^\infty \frac{e(t, \sqrt{\zeta})}{e(\zeta)} z(t)dt, \; \zeta \in \rho(T).$$

Denote the usual maximal operator by $M_{max}y = -y'' + q(x)y$, where $-y'' + q(x)y \in L^2(0, \infty)$, $y \in L^2(0, \infty)$. Denote

$$(M_{max,\zeta}z)(x) = \int_\infty^x k(x, t, \zeta)z(t)dt,$$

where

$$k(x, t, \zeta) = \frac{1}{2i\sqrt{\zeta}}[e(x, \sqrt{\zeta})e(t, -\sqrt{\zeta}) - e(x, -\sqrt{\zeta})e(t, \sqrt{\zeta})].$$

Note that if $F(u)$ is analytic in u, then $\frac{1}{\sqrt{\zeta}}[F(\sqrt{\zeta}) - F(-\sqrt{\zeta})]$ is analytic in ζ too.

Theorem. *Operator $M_{\max}$ is maximal for the operator M in the sense of Definition 2. Branching of resolvent is*

$$(M_\zeta z)(x) = \Big(\int_0^\infty s(t, \zeta)z(t)dt \frac{e(x, \sqrt{\zeta})}{e(\sqrt{\zeta})} + M_{max,\zeta}z \Big)(x).$$

$$\frac{e(x, \sqrt{\zeta})}{e(\sqrt{\zeta})} = s(x, \zeta)\frac{e'_x(\sqrt{\zeta})}{e(\sqrt{\zeta})} + C(x, \zeta).$$

1. M. A. Naimark, *Linear differential operators*, Moscow, Nauka, 1969.

Inverse problem for generalized de Branges matrices

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J-inner matrix valued functions appear in analysis as resolvent matrices of classical interpolation problems. Generalized J-inner matrices were introduced by V. Derkach, H. Dym in 2009 and serve as resolvent matrices of some indefinite interpolation problems. In the present talk we introduce the notion of entire de Branges matrix with negative index κ ($\kappa \in \mathbb{N} \cup \{0\}$) and consider the following completion problem:

Given an entire $p \times 2p$ de Branges matrix $\mathfrak{E}(\lambda)$ of negative index κ, find an entire $2p\times 2p$ generalized J-inner matrix $B(\lambda)$ such that $\mathfrak{E}(\lambda)=\sqrt{2}\begin{bmatrix}0 & I_p\end{bmatrix} B(\lambda)$.

In the case $\kappa = 0$ such a problem was considered by D. Arov and H. Dym in 2008.

Ateb-transforms and generalized shift operators

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In the 60s of the last century new special functions $ca(m, n, \omega)$ and $sa(m, n, \omega)$ called Ateb-functions (Ateb inverse Beta) were introduced. Ateb-functions have series expansions over all areas where the usual trigonometric functions do. We constructed Ateb-transform as a special type of Fourier transform. Using the theory of generalized shift operators (GSO) the algebra under the Hilbert functional space was constructed. The algebra contains "addition" and "multiplication" operations. The addition is the usual addition of functions (correctness follows from the additivity of the addition), and multiplication is a convolution. Since the periodic Ateb-functions are orthonormal, we can build a decomposition in a generalized Fourier series and build for them a generalized harmonic analisys by analogy.

We constructed hypergroup using GSO for Ateb-transforms and the corresponding convolution, completing within hypergroup algebra apparatus spectral and time analysis of functional spaces with the basis of Ateb-functions.

A note on the triadjoint of biorthomorphisms

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In this talk, we are interested in the Arens triadjoint of biorthomorphisms on Archimedean f-algebras.

1. K. Boulabair and W. Brahmi, *Multiplicative structure of biorthomorphisms and embedding of orthomorphisms*, Indag. Math. **27** (2016), 786–798.
2. R. Yilmaz, *The triadjoints of some bilinear maps*, Filomat, **28**:5 (2014), 963–979.
3. G. Buskes and R. Yilmaz, *A note on biorthomorphisms*, Operator Theory Advances and Appl. **201** (2009), 99–107.

On Schrödinger operators with singular rank-two perturbations

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In this talk we discuss the norm resolvent convergence as $\varepsilon \to 0$ of the family of Schrödinger operators

$$T_\varepsilon = -\frac{d^2}{dx^2} + \varepsilon^{-3} Q_\varepsilon + \varepsilon^{-1} q(\varepsilon^{-1} x), \qquad \operatorname{dom} T_\varepsilon = W_2^2(\mathbb{R})$$

where Q_ε is a rank-two operators acting in $L_2(\mathbb{R})$ as follows

$$(Q_\varepsilon v)(x) = f(\varepsilon^{-1} x) \int_{\mathbb{R}} g(\varepsilon^{-1} t) v(t)\, dt + g(\varepsilon^{-1} x) \int_{\mathbb{R}} f(\varepsilon^{-1} t) v(t)\, dt.$$

Our purpose is to find the limit operator and construct the so-called solvable models in terms of point interactions describing with admissible fidelity the real quantum interactions governed by the Hamiltonian T_ε. A careful analysis of T_ε leads us to five cases of the norm resolvent limits. The limiting behaviour is governed primarily by the functions f and g as well as their interaction with the potential q.

Let $h^{(-k)}$ be the k-fold primitive

$$h^{(-k)}(x) = \frac{1}{(k-1)!} \int_{-1}^{x} (x-t)^{k-1} h(t)\, dt$$

of function h. Without loss of generality we can assume that the supports of f, g and q lie in the interval $(-1,1)$. We introduce the notation

$$f_0=\int_{\mathbb{R}} f(t)\,dt,\ f_1=-\int_{\mathbb{R}} tf(t)\,dt,\ g_0=\int_{\mathbb{R}} g(t)\,dt,\ g_1=-\int_{\mathbb{R}} tg(t)\,dt,$$

$$n_f=\|f^{(-1)}\|_{L_2(-1,1)},\quad n_g=\|g^{(-1)}\|_{L_2(-1,1)},\quad p_{fg}=\int_{-1}^{1} f^{(-1)}\,g^{(-1)}\,dt,\quad (1)$$

$$\varkappa=n_gf_1-n_fg_1,\quad a_k=\int_{\mathbb{R}}\left(n_gf^{(-2)}-n_fg^{(-2)}\right)^k q\,dt,\ k=0,1,2.$$

The next theorem deals with the only case when the limit operators corresponds to non-trivial point interactions. This case is of special interest in the scattering theory.

Theorem. *Let f, g and q be integrable, real-valued functions with compact support contained in $(-1,1)$. Suppose that f and g have zero means, i.e., $f_0=g_0=0$. If $n_fn_g-p_{fg}=1$ and $a_2-\varkappa a_1\neq 0$, then the operators T_ε converge in the norm resolvent sense as $\varepsilon\to 0$ to the free Schrödinger operator $-\frac{d^2}{dx^2}$ restricted to functions in $W_2^2(\mathbb{R}\setminus\{0\})$ obeying the interface conditions*

$$\begin{pmatrix} v(+0)\\ v'(+0)\end{pmatrix}=\frac{1}{a_2-\varkappa a_1}\cdot\begin{pmatrix}\varkappa^2a_0-2\varkappa a_1+a_2 & \varkappa^2\\ a_0a_2-a_1^2 & a_2\end{pmatrix}\begin{pmatrix} v(-0)\\ v'(-0)\end{pmatrix}.$$

We see that the limit operator describes a wide class of point interactions at the origin.

1. Yu. Golovaty, *1D Schrödinger operators with short range interactions: two-scale regularization of distributional potentials*, Integral Equations Oper. Theory **75**:3 (2013),341–362.
2. Yu. Golovaty and R. Hryniv, *Norm resolvent convergence of singularly scaled Schrödinger operators and δ'-potentials.* Proc. R. Soc. Edinb. A., **143** (2013), 791–816.

On Sturm-Liouville operators with singular potentials

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We study the properties of Sturm-Liouville operators with distributional potentials in Hilbert space $L_2([a,b],\mathbb{C})$ given by homogeneous two-point boundary conditions. In particular, these potentials may be Radon measures.

We obtain sufficient conditions for these operators to be approximated by the sequence of operators with smooth coefficients in the sense of norm resolvent convergence. In the symmetric case we also investigate coapproximation (that is, approximation by operators of the same class) of self-adjoint, maximal dissipative or maximal accumulative operators.

1. A. S. Goriunov and V.A. Mikhailets, *Regularization of singular Sturm-Liouville equations*, Meth. Funct. Anal. Topol. **16**:2 (2010), 120–130.
2. A. S. Goriunov, V. A. Mikhailets, and K. Pankrashkin, *Formally self-adjoint quasi-differential operators and boundary-value problems*, Electron. J. Diff. Equ. **101** (2013), 1–16.
3. A. S. Goriunov, *Convergence and approximation of the Sturm-Liouville operators with potentials-distributions*, Ukr. Math. J. **67**:5 (2015), 680–689.

Reconstruction of singular quantum trees from spectral data

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We discuss the inverse spectral theory for singular quantum trees, i.e., for differential operators with singular coefficients acting on metric trees. In particular, we study the inverse problem of reconstructing the singular Sturm–Liouville operator on a metric star-type graph with n edges from $n+1$ spectra.

Furthermore, we investigate the situation when the potential on one or several edges of the star-type graph is known and find out what spectral information is sufficient to reconstruct the potential on the whole graph.

1. M. Homa and R. Hryniv, *Inverse problems for singular quantum trees*, (submitted).

On convergence of SP iteration scheme in hyperbolic space

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The present study aims to deal with multivalued version of SP iterative scheme to approximate a common fixed point of three multivalued nonexpansive mappings in a uniformly convex hyperbolic space and obtain strong and Delta-convergence theorems for the SP process.

A truncated indefinite Stieltjes moment problem

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A function f meromorphic on $\mathbb{C}\backslash\mathbb{R}$ with the set of holomorphy $\mathfrak{h}_f$ is said to be in the generalized Nevanlinna class $\mathbf{N}_\kappa$ ($\kappa \in \mathbb{N}$), if for every set $z_i \in \mathbb{C}_+ \cap \mathfrak{h}_f$ ($j = 1, \ldots, n$) the form

$$\sum_{i,j=1}^{n} \frac{f(z_i) - \overline{f(z_j)}}{z_i - \bar{z}_j} \xi_i \bar{\xi}_j$$

has at most κ and for some choice of z_i ($i = 1, \ldots, n$) it has exactly κ negative squares. For $f \in \mathbf{N}_\kappa$ let us write $\kappa_-(f) = \kappa$.

A function $f \in \mathbf{N}_\kappa$ is said to belong to the class $\mathbf{N}_\kappa^k$ ($k \in \mathbb{N}$) if $zf \in \mathbf{N}_\kappa^k$ (see [1]). If $\kappa = k = 0$ the class $\mathbf{N}_0^0$ coincides with the Stieltjes class introduced by M.G. Krein in 1952.

Problem. $\mathbf{MP}_\kappa^k(\mathbf{s}, \ell)$. Given ℓ, κ, $k \in \mathbb{Z}_+$, and a sequence $\mathbf{s} = \{s_i\}_{i=0}^{\ell}$ of real numbers, describe the set $\mathcal{M}_\kappa^k(\mathbf{s})$ of functions $f \in \mathbf{N}_\kappa^k$, which have the following asymptotic expansion

$$f(z) = -\frac{s_0}{z^1} - \frac{s_1}{z^2} - \cdots - \frac{s_\ell}{z^{\ell+1}} + o\Big(\frac{1}{z^{\ell+1}}\Big), \quad z\widehat{\rightarrow}\infty.$$

The set $\mathcal{N}(\mathbf{s}) = \{n_j\}_{j=1}^{N}$ of normal indices of the sequence $\mathbf{s}$ is defined by

$$\mathcal{N}(\mathbf{s}) = \{n_j : D_{n_j} \neq 0, j = 1, 2, \ldots, N\}, \quad D_{n_j} := det(s_{i+k})_{i,k=0}^{n_j-1}.$$

Let $D_n^+ := det(s_{i+j+1})_{i,j=0}^{n-1}$. It is shown that $\mathcal{N}(\mathbf{s})$ is the union of two not necessarily disjoint subsets $\mathcal{N}(\mathbf{s}) = \{\nu_j\}_{j=1}^{N_1} \cup \{\mu_j\}_{j=1}^{N_2}$, which are selected by

$$D_{\nu_j} \neq 0 \quad \text{and} \quad D_{\nu_j-1}^+ \neq 0, \qquad \text{for all } j = \overline{1, N_1},$$

$$D_{\mu_j} \neq 0 \quad \text{and} \quad D_{\mu_j}^+ \neq 0, \qquad \text{for all } j = \overline{1, N_2}.$$

Moreover, the normal indices ν_j and μ_j satisfy the following inequalities

$$0 < \nu_1 \leqslant \mu_1 < \nu_2 \leqslant \mu_2 < \ldots$$

Theorem. *Let $N \in \mathbb{N}$. Then any solution of the moment problem $MP_\kappa^k(\mathbf{s}, 2\nu_N - 2)$ admits the following representation*

$$f(z) = \frac{Q_{2N-1}^+(z)\tau(z) + Q_{2N-2}^+(z)}{P_{2N-1}^+(z)\tau(z) + P_{2N-2}^+(z)},$$

where $P_i^+(z)$ and $Q_i^+(z)$ are generalized Stieltjes polynomials of the first and the second kind, respectively, and parameter $\tau(z)$ satisfies the following conditions

$$\tau \in \mathbf{N}_{\kappa-\kappa_N}^{k-k_N} \quad \text{and} \quad \frac{1}{\tau(z)} = o(z), \quad z\widehat{\to}\infty.$$

Finite rank perturbations of finite gap Jacobi and CMV operators

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Necessary and sufficient conditions are presented for a measure to be the spectral measure of a finite rank perturbation of a Jacobi or CMV operator from a finite gap isospectral torus. As special cases, this includes finite rank perturbations of Jacobi/CMV operators with constant and periodic coefficients.

We also solve the inverse resonance problem: it is shown that an operator is completely determined by the set of its eigenvalues and resonances, and we provide necessary and sufficient conditions on their configuration for such an operator to exist.

Paper reference: (**http://arxiv.org/abs/1410.7272**).

Trace formula for discrete Schrodinger operators

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In the Hilbert space $\ell_2(\mathbb{Z})$, we consider a discrete Schrodinger operator T_q defined by the expression

$$(T_q f)(n) = f(n+1) + f(n-1) + q(n)f(n), \qquad n \in \mathbb{Z},$$

with the potential q belonging to $\ell_1(\mathbb{Z})$. In the talk, the trace formula for the operator T_q will be proved.

On general parabolic problems in Hörmander spaces

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We discuss applications of Hörmander inner product spaces to $2b$-parabolic linear differential equations [1, 2, 3, 4]. These spaces are parametrized with a pair of real numbers and a Borel measurable function that varies slowly at infinity in the sense of Karamata. We introduce a class of anisotropic inner product Hörmander spaces on a smooth lateral cylinder surface. These spaces do not depend on the choice of special local coordinates on the surface. All these spaces can be obtained by interpolation with a function parameter between anisotropic Sobolev spaces.

We proved that the operators corresponding to a general parabolic problem are isomorphisms between appropriate Hörmander spaces. We also proved versions of this isomorphism theorem for homogeneous initial conditions and parabolic systems. We discuss applications of these theorems to investigation of local regularity of generalized solutions to parabolic problems. Specifically, we find new sufficient conditions under which these solutions are classical.

1. V. Los, V. A. Mikhailets, and A. A. Murach, *An isomorphism theorem for parabolic problems in Hörmander spaces and its applications*, Communs Pure and Appl. Anal. **16**:1 (2017), 69–97.
2. V. Los and A. Murach, *Isomorphism theorems for some parabolic initial-boundary value problems in Hörmander spaces*, Open Math. **15** (2017), 57–76.
3. V. Los, *Sufficient conditions for the solutions of general parabolic initial-boundary-value problems to be classical*, Ukr. Math. J. **68**:11 (2017), 1756–1766.
4. V. Los, *Anisotropic Hörmander spaces on the lateral surface of a cylinder*, J. Math. Sci. (NY) **217**:4 (2016), 456–467.

Operators of generalized differentiation of infinite order in spaces of type S

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Let $S_{l_k}^{m_n}$ be the set of all functions $\varphi \in \mathbb{C}^\infty(\mathbb{R})$ satisfying the condition:

$$\exists c > 0 \exists A > 0 \exists B > 0 \forall \{k, n\} \subset \mathbb{Z}_+ \forall x \in \mathbb{R} : |x^k \varphi^{(n)}(x)| \leq cA^k B^n l_k m_n$$

(const c, A, $B > 0$ depend on the function φ). Here $\{l_k, k \in \mathbb{Z}_+\}$, $\{m_n, n \in \mathbb{Z}_+\}$ are monotonically increasing sequences of positive numbers satisfying the conditions:

1) $\forall n \in \mathbb{Z}_+$: $m_n \le m_{n+1}$; $m_0 = 1$;
2) $\forall \alpha > 0 \exists c_\alpha > 0 \forall n \in \mathbb{Z}_+$: $m_n \ge c_\alpha \cdot \alpha^n$;
3) $\exists M > 0\ \exists h > 0\ \forall n \in \mathbb{Z}_+$: $m_{n+1} \le M h^n m_n$.

Theses spaces are generalizations of the spaces S_α^β [1], which are studied in detail in the most detail when $l_k = k^{k\alpha}$, $m_n = n^{n\beta}$, where α, $\beta > 0$ are fixed parameters.

We consider the sequences $\{m_n = n!\rho_n, n \in \mathbb{Z}_+\}$ and $\{l_k = k!d_k,\ k \in \mathbb{Z}_+\}$, where the sequences $\{\rho_n, n \in \mathbb{Z}_+\}$ and $\{d_k, k \in \mathbb{Z}_+\}$ are strinctly decreasing and satisfy the condition $\lim\limits_{n\to\infty} \sqrt[n]{\alpha_n} = 0$ [2].

Theorem 1. *The function $\varphi \in C^\infty(\mathbb{R})$ belongs to the set $S_{l_k}^{m_n}$ if and only if it analytically extends to an entire function $\varphi(z)$, $z \in \mathbb{C}$, which satisfy the condition*

$$\exists a > 0 \exists b > 0 \exists c > 0 \forall z = x + iy \in \mathbb{C} : |\varphi(z)| \le c\gamma(ax)\rho(by),$$

where

$$\gamma(x) = \begin{cases} 1, & |x| < 1, \\ \inf\limits_{k\in\mathbb{Z}_+} (l_k/|x|^k), & |x| \ge 1, \end{cases} \quad \rho(y) = \begin{cases} 1, & |y| < 1, \\ \sup\limits_{n\in\mathbb{Z}_+} (|y|^n/m_n), & |y| \ge 1. \end{cases}$$

In the spaces $S_{m_k}^{m_n}$ consider linear continuous operators $g(A) = \sum\limits_{k=0}^{\infty} c_k A^k$, where $A = D^n(F, \cdot)$, $n \in \mathbb{N}$, which are treated as operators of generalized differentiation of infinite order. We note that $D^n(F,\varphi)(x) := \sum\limits_{k=n}^{\infty} b_k \frac{a_{k-n}}{a_k} x^{k-n}$ is the operator of the generalized differentiation of Gel'fond-Leontiev, where $\{a_k,\ k \in \mathbb{Z}_+\}$, $\{b_k,\ k \in \mathbb{Z}_+\}$ are the sequence of Taylor coefficients of the functions F and φ respectively.

Theorem 2. *If the entire function $g(z) = \sum\limits_{m=0}^{\infty} c_m z^m$, $z \in$ C, satisfies the condition*

$$\exists a > 0\, \exists b > 0\, \exists c > 0\, \forall z = x + iy \in \mathbb{C} :\ |g(z)| \le c\rho(ax)\rho(by),$$

than the operator $A_g := g(D(F,\cdot))$ is defined in space $S_{m_k}^{m_n}$:

$$g(D(F,\psi))(x) = \sum_{m=0}^{\infty} c_m D^m(F,\psi)(x), \quad x \in \mathbb{R}, \psi \in S_{m_k}^{m_n},$$

which continuously maps the space $S_{m_k}^{m_n}$ to $S_{m_k}^{m_n}$.

1. I. M. Gelfand and G. E. Shylov, *Spaces of main and generalized functions*, Fizmatgiz, Moscow, 1958. 307 pp. (in Russian).

2. V. V. Gorodetsky and O. V. Martynyuk, *Gelfond-Leontiev generalized differentiation operators in spaces of type S*, Sib. Math. J. **54**:3 (2013), 446–458.

On Hörmander spaces and interpolation

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The talk is a survey of our results [1, 2, 3, 4, 5] devoted to Hörmander spaces and their application to interpolation between Sobolev spaces. We discuss interpolation properties of a class of inner product Hörmander spaces parametrized with a radial function parameter which is OR-varying at $+\infty$. This parameter characterizes regularity of distributions in terms of the behaviour of their Fourier transform at infinity. These spaces are considered over $\mathbb{R}^n$ or a half-space in $\mathbb{R}^n$ or a bounded Euclidean domain with Lipschitz boundary. The class of these spaces has the following interpolation properties: it consists of all interpolation Hilbert spaces between inner product Sobolev spaces, is obtained by the interpolation with function parameter between inner product Sobolev spaces, and is closed with respect to the interpolation with function parameter between Hilbert spaces. We introduce the corresponding class of Hörmander inner product spaces over an arbitrary closed infinitely smooth manifold. These spaces and the topology in them do not depend on the choice of local charts and partition of unity on the manifold (the choice is used in the local definition of these spaces). This class has the interpolation properties mentioned above. They allow us to build the theory of solvability of general elliptic systems (on manifolds) and elliptic boundary-value problems in Hörmander spaces considered in [1, 3, 4, 6, 7].

1. V. A. Mikhailets and A. A. Murach, *Hörmander Spaces, Interpolation, and Elliptic Problems*, De Gruyter, Berlin/Boston, 2014, xii+297 pp. (Russian version is available at arXiv:1106.3214).
2. V. A. Mikhailets and A. A. Murach, *Interpolation with a function parameter and refined scale of spaces*, Methods Funct. Anal. Topology **14**:1 (2008), 81–100.
3. V. A. Mikhailets and A. A. Murach, *Elliptic problems and Hörmander spaces*, Operator Theory Adv. Appl. **190** (2009), 455–478 (arXiv:0904.0372).
4. V. A. Mikhailets and A. A. Murach, *The refined Sobolev scale, interpolation, and elliptic problems*, Banach J. Math. Anal. **6**:2 (2012), 211–281 (arXiv:1206.6041).
5. V. A. Mikhailets and A. A. Murach, *Interpolation Hilbert spaces between Sobolev spaces*, Results Math. **67**:1 (2015), 135–152 (arXiv:1106.2049).
6. A. V. Anop and A. A. Murach, *Regular elliptic boundary-value problems in the extended Sobolev scale*, Ukr. Math. J. **66**:7 (2014), 969–985 (arXiv:1310.4111).
7. T. N. Zinchenko and A. A. Murach, *Petrovskii elliptic systems in the extended Sobolev scale*, J. Math. Sci. (New York), **196** (2014), 721–732.

Spectral properties of the Hill-Schrödinger operators with distributional potentials

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We give review of results [1-7] on spectral properties of the Hill-Schrödinger operators with distributional potentials. Main attention is paid to the properties of the spectral gaps. In particular, we investigate the connection between the asymptotic behavior of the sequence of Fourier coefficients of the potentials in the corresponding sequence spaces and the lengths of the spectral gaps.

1. V. Mikhailets and V. Molyboga, *One-dimensional Schrödinger operators with singular periodic potentials*, Meth. Funct. Anal. Topol. **14**:2 (2008), 184–200.
2. V. Mikhailets and V. Molyboga, *Spectral gaps of the one-dimensional Schrödinger operators with singular periodic potentials*, Meth. Funct. Anal. Topol. **15**:1 (2009), 31–40.
3. V. Mikhailets and V. Molyboga, *Hill's potentials in Hörmander spaces and their spectral gaps*, Meth. Funct. Anal. Topol. **17**:3 (2011), 235–243.
4. V. Mikhailets and V. Molyboga, *Smoothness of Hill's potential and lengths of spectral gaps*, OT: Adv. & Appl. **221** (2012), 467–478.
5. V. Mikhailets and V. Molyboga, *Spectral gaps of the Hill-Schrödinger operators with distributional potentials*, Meth. Funct. Anal. Topol. **20**:4 (2014), 321–327.
6. V. Molyboga, *Characterization of the gaps in the spectrum of the Hill operator with distribution as potential*, Proc. Inst. of Math. NASU **10**:2 (2013), 248–259.
7. V. Molyboga, *About boundedness of solutions of the Hill equation with distribution potential*, Proc. Inst. of Math. NASU **11**:2 (2014), 274–283.

Block-Jacobi operators: the spectrum and asymptotic properties of generalized eigenvectors

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So-called generalized eigenvectors play an important role in studies of scalar and block-Jacobi operators ("matrices"). And to study generalized eigenvectors it is convenient to rewrite the formal eigenequation in the "linear recursion" form, i.e. in the general form

$$x_{n+1} = A_n x_n, \quad n \geq n_0,$$

where $\{A_n\}_{n\geq n_0}$ is some sequence of matrices and x_n-s are vectors. In our case of generalized eigenvectors of block-Jacobi operators, with d being the dimension of blocks ($d = 1$ in scalar case), the role of $\{A_n\}_{n\geq n_0}$ is played by the $2d\times 2d$ dimensional "transfer matrix" sequence (with the spectral parameter $\lambda \in \mathbb{R}$), and

$$x_n := \begin{pmatrix} u_{n-1} \\ u_n \end{pmatrix},$$

where $\{u_n\}_{n\geq 1}$ is just generalized eigenvector for λ. — Denote the set of all solutions $\{x_n\}_{n\geq 2}$ of such linear recursion by $\mathrm{Sol}(\lambda)$ for block Jacobi operator J and fixed spectral parameter λ.

For the scalar-Jacobi case it is known that there is a close relation between some asymptotic properties of all sequences from $\mathrm{Sol}(\lambda)$ and spectral properties of J (e.g., the Subordination Theory by Gilbert-Pearson-Khan).

Here we consider some new questions concerning such relations also for block-Jacobi operators. We also show some abstract results on asymptotic properties of solutions of linear recursions.

Transformation operators for Jacobi operators

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In the Hilbert space $\ell_2(\mathbb{Z})$, we consider a self-adjoint Jacobi operator T generated by the expression

$$(Tf)(n) = a(n)f(n+1) + a(n-1)f(n-1) + b(n)f(n), \qquad n \in \mathbb{Z},$$

where $(a_n)_{n\in\mathbb{Z}}$ and $(b_n)_{n\in\mathbb{Z}}$ belong to $\ell_\infty(\mathbb{Z})$. In the talk, conditions under which there exist transformation operators for the operator T will be studied.

On “all but m” families of projections

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In [1], the structure of families of projections, $Q_1, \dots, Q_n$, P, for which $Q_1 + \dots + Q_n = I$, has been studied. Such families arise, in particular, in the study of Toeplitz operators with piecewise continuous symbols. It was shown that such families can be described in terms of families of positive self-adjoint operators $C_1, \dots, C_n$, for which $C_1 + \dots + C_n = I$. In the case, where these operators form a commuting family, the irreducible representations correspond to the points of a simplex $c_1 + \dots + c_n = 1$, $c_j \geq 0$, $j = 1, \dots, n$.

Representations of the $*$-algebra generated by m projections $P_1, P_2, \dots, P_m$, $P_i \perp P_j$, $i, j = 1, \dots, m$ and n more projections $Q_1 + \dots + Q_n = I$ in a Hilbert space arise in the study of a wider class of Toeplitz operators [2]. We show that representations of such algebra can be described in terms of families of positive self-adjoint $m \times m$ block matrices $B_1, \dots, B_n$, for which $B_1 + \dots + B_n = I$. Also, we construct a family of normal operators C_α where α is a multi-index, such that the commutativity of the family of all C_α is equivalent to $\dim P_j \leq 1$, $j = 1, \dots, m$ for irreducible representations. A finite subset of multi-indices is selected, which is sufficient to identify an irreducible representation.

1. N. Vasilevski, *C^*-algebras generated by orthogonal projections and their applications*, Int. Equat. Oper. Theory. **31** (1998), 113–132.
2. Yu. I. Karlovich and L. V. Pessoa, *C^*-algebras of Bergman type operators with piecewise continuous coefficients*, Int. Equat. Oper. Theory. **57** (2007), 521–565.

Defining power of the compositions involving Dirac-delta and infinitely differentiable functions

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Let f be an infinitely differentiable function. The distribution $\delta(f(x))$ is defined as the neutrix limit of the regular sequence $\{\delta_n(f(x))\}$, where $\{\delta_n(x)\}$ is a regular sequence of infinitely differentiable functions converging to the Dirac-delta function. In this talk we give meaning to the k-th powers of the distribution $\delta(f(x))$ for an infinitely differentiable function f having a simple root and we further assume that $f = g^s$ where g has a simple single root. Additionally we consider the particular case $\delta^k(x_+^\lambda)$ for $\lambda > 0$, $k = 1, 2, \dots$.

On linear combinations of four orthogonal projections

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We give a new proof that every bounded self-adjoint operator in a Hilbert separable space is a real linear combination of 4 orthoprojections. We prove that not every self-adjoint operator of the form scalar plus compact is a real linear combination of three orthoprojections, which has been an open problem since 1984. Also we show that there are bounded operators that can not be decomposed into a complex linear combination of four orthoprojections. Using ideas applied in infinite dimensional space, we find $n \times n$ matrices that are not real linear combinations of 3 orthoprojections for every $n \geq 76$.

Spectrum, trace and oscillation of a Sturm-Liouville type retarded differential operator with transmission conditions

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In this study, a formula for regularized sums of eigenvalues of a Sturm-Liouville problem with retarded argument at the point of discontinuity is obtained. Moreover, oscillation properties of the related problem is investigated.

On solvable extensions of some nondensely defined operators in Hilberrt space

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In this report the role of initial object (L, L_0) is played by a couple of closed densely defined linear operators acting in a fixed complex Hilbert space H equipped with the inner product $(\cdot|\cdot)$, such that $L_0 \subset L$. We use the following notations: $(\cdot|\cdot)_X$ is an inner product in a Hilbert space X; $D(T)$, $R(T)$, $ker\, T$, $\rho(T)$ are, respectively, the domain, range, kernel and the resolvent set of a relation (in particular, operator) T; T^* is the adjoint of T; $B(X, Y)$ is the set of linear bounded operators $A : X \to Y$ such that $D(A) = X$ (see [1] for the details) and $D[T]$ means $D(T)$ considered as a Hilbert space equipped with the graph inner product of T.

Put $M = L_0^*$, $M_0 = L^*$. It is known [2] that there exist Hilbert spaces G_1, G_2 and operators $\Gamma_i \in B(D[L], G_i)$, $i \in \{1, 2\}$, such that

$$R(\Gamma_1 \oplus \Gamma_2) = G_1 \oplus G_2, \qquad \ker(\Gamma_1 \oplus \Gamma_2) = D(L_0)$$

and the operators $\tilde{\Gamma}_1 \in B(D[M], G_2)$, $\tilde{\Gamma}_2 \in B(D[M], G_1)$, uniquely determinated by $G_1, G_2, \Gamma_1, \Gamma_2$, such that

$$R(\tilde{\Gamma}_1 \oplus \tilde{\Gamma}_2) = G_2 \oplus G_1, \qquad \ker(\tilde{\Gamma}_1 \oplus \tilde{\Gamma}_2) = D(M_0)$$

and

$$\forall y \in D(L) \quad \forall z \in D(M) \quad (Ly|z) - (y|Mz) = (\Gamma_1 y|\tilde{\Gamma}_2 z)_{G_1} - (\Gamma_2 y|\tilde{\Gamma}_1 z)_{G_2}.$$

Further, let $H_0^{(L)}$, $H_0^{(M)}$ be finite-dimensional subspaces of H. Put

$$S_0 = L_0{\downarrow}(H\Theta H_0^{(L)}), \qquad T_0 = M_0 \downarrow (H\Theta H_0^{(M)}),$$

$$S = \left\{(y, Ly + \varphi^{(M)}) : y \in (L),\ \varphi^{(M)} \in H_0^{(M)}\right\},$$

$$T = \left\{(z, Mz + \varphi^{(L)}) : z \in D(M),\ \varphi^{(L)} \in H_0^{(L)}\right\}$$

(here and below $\downarrow$ is the symbol of restriction of mapping). Furthermore, denote by $P_0^{(L)}$, $P_0^{(M)}$, respectively, the orthoprojections $H \to H_0^{(L)}$, $H \to H_0^{(M)}$ and introduce the following notations:

$$G_{s,1} = G_1 \oplus H_0^{(L)},\ G_{s,2} = G_2 \oplus H_0^{(M)},\ G_s = G_{s,1} \oplus G_{s,2},$$

$$\Gamma_{S,1}(y, \varphi^{(M)}) = (\Gamma_1 y, -P_o^{(L)} y),$$

$$\Gamma_{S,2}(y,\varphi^{(M)}) = (\Gamma_2 y, -P_o^{(L)} y) \quad (y \in D(L), \varphi^{(M)} \in H_0^{(M)});\ \Gamma_s = \Gamma_{s,1} \oplus \Gamma_{s,2}.$$

Assume that F is a a complex Hilbert space such that $\dim F = \dim G_s$. It is not hard to prove that for each closed extension of relation S_0, being the restriction of S, there exists an operator $A \in B(G,F)$ such that $S_A = \ker A\Gamma_S$, where S_A is the mentioned extension. Consequently: $S_A = \ker(A_1\Gamma_{s,1} + A_2\Gamma_{S,2})$, where $A_i = A \downarrow G_{s,i}$, $i \in \{1,2\}$.

We suppose that the resolvent set $\rho(L_2)$ of the operator $L_2 := L{\downarrow} \ker \Gamma_2$ is not empty and $\lambda \in \rho(L_2)$. In this report the conditions of solvability of the relation $S_A - \lambda$ are investigated and the connection between the resolvents of S_A and L_2 is established.

1. R. Arens, *Operational calculus of linear relations*, Pacif. J. Math. **11**:1 (1961), 9–23.
2. Yu. I. Oliyar, O. G. Storozh, *Criteria of mutual adjointness of proper extensions of linear relations*, Math. Stud. **40**:1 (2013), 71–78.

Boundary triples for integral systems on the half-line

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Let functions P, Q and W be of locally bounded variation on $[0,\infty)$ and W be nondecreasing. The following integral system

$$J\vec{f}(x) - J\vec{a} = \int_0^x \begin{pmatrix} \lambda dW - dQ & 0 \\ 0 & dP \end{pmatrix} \vec{f}(t),\ J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

on the half-line $[0,\infty)$ has been studied in [1]. It contains the Sturm-Liouville equation, Stieltjes string and more general Krein-Feller string equations as special cases.

This integral system generates a maximal and a minimal linear relations A_{max} and A_{min} in the Hilbert space $L^2(W)$, such that A_{min} is symmetric and $A_{max} = A^*_{min}$.

It is shown that deficiency indices $n_\pm(A_{min})$ are either $(1,1)$ or $(2,2)$. In both cases boundary triples for A_{max} are constructed and the corresponding Weyl functions (see definitions in [2]) are calculated. The case when P, Q and W are of bounded variation on $[0,\infty)$ is called quasiregular. In this case the linear relation A_{min} is proved to have deficiency indices $(2,2)$ and the boundary triple for A_{max} can be written in a simpler form.

1. C. Bennewits, *Spectral asymptotics for Sturm-Liouville equations*, Proc. London Math. Soc. (3) **59**:2 (1989), 294–338.
2. V. A. Derkach and M. M. Malamud, *The extension theory of Hermitian operators and the moment problem*, J. Math. Sci. **73**:2 (1995), 141–242.

On topological conditions in the Marchenko theorem

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In the Hilbert space $L_2(\mathbb{R}_+)$, we consider the Schrödinger operator T_q generated by the differential expression

$$t_q(f) := -\frac{d^2}{dx^2} + q$$

and the boundary condition

$$f(0) = 0$$

with the potential q belonging to the Marchenko class

$$\mathcal{Q} := \{q \in L_1(\mathbb{R}_+, x\mathrm{d}x) \mid \operatorname{Im} q = 0\}.$$

In the class $\mathcal{T} := \{T_q \mid q \in \mathcal{Q}\}$, the inverse scattering problem has a unique solution that was found by V. A. Marchenko [1] who proved a theorem providing a complete description of the scattering data for the operators $T \in \mathcal{T}$. In this talk, we will show that topological conditions in this theorem can be replaced by the condition that the scattering function should belong to a certain Banach algebra.

1. V. A. Marchenko, *Sturm-Liouville Operators and their Applications*, (Kiev: Naukova Dumka) (in Russian) (Engl. transl. 1986 (Basel: Birkhäuser)).

Spectral properties of Sturm–Liouville equations with singular energy-dependent potentials

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The aim of the talk is to discuss spectral properties of Sturm–Liouville problems with energy-dependent potentials given by the differential equations

$$-y'' + qy + 2\lambda py = \lambda^2 y \tag{1}$$

on (0,1) subject to the Dirichlet boundary conditions

$$y(0) = y(1) = 0. \tag{2}$$

Here p is a real-valued function from $L_2(0,1)$, q is a real-valued distribution from the Sobolev space $W_2^{-1}(0,1)$, and $\lambda \in \mathbb{C}$ is a spectral parameter.

Denote by A the operator acting via $Ay := \ell(y)$ on the domain $\operatorname{dom} A := \{y \in \operatorname{dom} \ell \mid y(0) = y(1) = 0\}$. Next we denote by B the operator of multiplication by the function $2p \in L_2(0,1)$, by I the identity operator, and define the quadratic operator pencil T as

$$T(\lambda) := \lambda^2 I - \lambda B - A, \qquad \lambda \in \mathbb{C},$$

on the λ-independent domain $\operatorname{dom} T := \operatorname{dom} A$. Then the spectral problem (1)–(2) can be regarded as the spectral problem for the operator pencil T. Our main result is the following

Theorem. *Let κ be the number of negative eigenvalues of the operator A. Then*

- *the spectrum of the problem* (1), (2) *is discrete;*
- *there are at most κ non-simple real eigenvalues in the spectrum of* (1), (2)*, and their algebraic multiplicities do not exceed $2\kappa + 1$;*
- *the non-real spectrum of* (1), (2) *is symmetric with respect to the real axis and consists of at most κ pairs of eigenvalues λ and $\bar{\lambda}$ of finite algebraic multiplicity, moreover, the root subspaces corresponding to λ and $\bar{\lambda}$ are isomorphic.*

If the operator A is positive, then the spectrum of (1), (2) *is real and simple.*

To prove this theorem we use linearization of the operator pencil T and its properties in the corresponding Pontryagin space.

1. N. Pronska. *Spectral properties of Sturm–Liouville equations with singular energy-dependent potentials*, Methods Funct. Anal. Topol. **19**:4 (2013), 327–345.

Model representation of quadratic operator pencils

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We derive model representation of non-selfadjoint quadratic operator pencils in terms of spectral resolution of the corresponding operator roots. The models are given in terms of Hilbert or Stieltjes transforms.

Section: Topology and Topological Algebra

On compatible group topologies on LCA groups

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The topology on a LCA group G is the finest locally quasi–convex group topology which gives rise to the character group $G^\wedge$ while the Bohr topology $\sigma(G, G^\wedge)$ is the coarsest topology with this property. For compact abelian groups both topologies coincide. A locally quasi–convex group topology is called compatible with the original topology if it has the same character group.

In the talk concrete examples for compatible topologies on $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{Z}(p^\infty)$ and on infinite products of discrete abelian groups will be given. Further, some properties of the poset $\mathcal{C}(G)$ of all compatible group topologies on G will be presented. If G is an infinite product of discrete groups, then $|\mathcal{C}(G)| = 2^{2^{|G|}}$ is as big as possible, and $|\mathcal{C}(\mathbb{R})| = \mathfrak{c}$, $|\mathcal{C}(\mathbb{Z})| = \mathfrak{c}$, and $|\mathcal{C}(\mathbb{Z}(p^\infty))| = \mathfrak{c}$ hold.

A quantitative generalization of Prodanov-Stoyanov Theorem on minimal Abelian topological groups

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A topological group X is defined to have *compact exponent* if for some number $n \in \mathbb{N}$ the set $\{x^n : x \in X\}$ has compact closure in X. Any such number n is called a *compact exponent* of X.

Our principal result states that a complete Abelian topological group X has compact exponent (equal to $n \in \mathbb{N}$) if and only if for any injective continuous homomorphism $f : X \to Y$ to a topological group Y and every $y \in \overline{f(X)}$ there exists a positive number k (equal to n) such that $y^k \in f(X)$.

This result has many interesting implications:

- an Abelian topological group is compact if and only if it is complete in each weaker Hausdorff group topology;
- each minimal Abelian topological group is precompact (this is the famous Prodanov-Stoyanov Theorem);
- a topological group X is complete and has compact exponent if and only if it is closed in each Hausdorff paratopological group containing X as a topological subgroup (this confirms an old conjecture of Banakh and Ravsky).

Paper reference: (**http://arxiv.org/abs/1706.05411**).

Topological graph inverse semigroups

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We investigate a topologization of λ-polycyclic monoids and graph inverse semigroups. In particular, we prove that a locally compact Hausdorff topological λ-polycyclic monoid is discrete. We describe all locally compact Hausdorff topologies which make λ-polycyclic monoid a semitopological semigroup.

Our main result is a complete characterization of graph inverse semigroups which admit only discrete locally compact Hausdorff semigroup topology. This characterization provides a complete answer on the question of Z. Mesyan, J.D. Mitchell, M. Morayne and Y.H. Péresse.

Paper reference: (**http://arxiv.org/abs/1706.08594**).

On scatteredly continuous functions

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We shall discuss some new results and open problems related to the scatteredly and weak continuity of functions.

On the knot digraphs and its bitopologies

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A knot is an embedding of a circle S^1 in 3-dimension Euclidean space $\mathbb{R}^3$ (or S^3) , considered up to continuous deformations (isotopies). A knot in $\mathbb{R}^3$ (respectively in the 3-sphere, S^3), can be projected onto a plane $\mathbb{R}^2$ (resp. a sphere S^2). This projection is almost always regular, meaning that it is injective everywhere, except at a finite number of crossing points, which are the projections of only two points of the knot, and these points are not collinear. Here we focus on Seifert graphs, which are the ribbon graphs of a knot diagram that arise from Seifert surface. In this talk bitopologies, which corresponds to the Seifert graphs of the $(2, n)$-Torus Knots, are found by using quasi-pseudo metric. Also, topological properties of them are analysed.

On the continuity on differentiable curves

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Recent investigations by A. Rosenthal [1] of the connections between the joint continuity of functions $f : \mathbb{R}^2 \to \mathbb{R}$ and their continuity on differentiable and twice differentiable curves were continued in [2, 3, 4]. On the other hand, in [5] the authors introduced the notion of Ω_{x_0}-continuity for a mapping $f : X \to Y$, where X and Y are topological spaces, $x_0 \in X$ and Ω_{x_0} is the set of all curves $\omega : [0,1] \to X$ with $\omega(0) = x_0$, which means that all the compositions $f \circ \omega$ are continuous at 0. The following result was obtained in [5].

Theorem 1. *Let C_{x_0} be the set of all continuous curves $\omega : [0,1] \to X$ with $\omega(0) = x_0$, where X is a topological vector space and Y is an arbitrary topological space. Then a function $f : X \to Y$ is C_{x_0}-continuous if and only if it is sequentially continuous at the point x_0.*

If X is a normed space, then by D_{x_0} we denote the subset of C_{x_0} consiting of all differentiable curves $\omega : [0,1] \to X$ with $\omega(0) = x_0$. The following theorem holds.

Theorem 2. *Let X be a normed space, Y a topological space and $x_0 \in X$. Then the D_{x_0}-continuity of a function $f : X \to Y$ is equivalent to its continuity at the point x_0.*

The proof is based on the next statement.

Lemma. *Let X be a normed space, a, b be points of X and $[\alpha, \beta]$ be a non-degenerated segment of the real line $\mathbb{R}$. Then the polynomial*

$$\omega(t) = \frac{(\beta - t)a + (t - \alpha)b}{\beta - \alpha} - \frac{(t - \alpha)(t - \beta)(2t - \alpha - \beta)}{(\beta - \alpha)^3}(b - a)$$

has $\omega(\alpha) = a$, $\omega(\beta) = b$, $\omega'(\alpha) = 0 = \omega'(\beta)$ and $||\omega(t)|| \le 3(||a|| + ||b||)$ on $[\alpha, \beta]$.

1. A. Rosenthal, *On the continuity of functions of several variables*, Math. Z. **63** (1955), 31–38.
2. K. Ciesielski and T. Glatzer, *Functions continuous on twice differentiable curves, discontinuous on large sets*, Real Anal. Exchange **37**:2 (2012), 353–361.
3. K. Ciesielski and T. Glatzer, *Sets of discontinuities of linearly continuous functions*, Real Anal. Exchange, **38**:2 (2013), 337–389.
4. K. Ciesielski and T. Glatzer, *Sets of discontinuities for functions continuous on flats*, Real Anal. Exchange, **39**:1 (2014), 117–138.

5. V. Maslyuchenko and O. Fotiy, *On sequentially continuous functions*, Bukovinian Math. J. **5**:1-2 (2017), 105–111 (in Ukrainian).

The generalized backward shift operator on $\mathbb{Z}[[x]]$, Cramer's formulas for solving infinite linear systems, and p-adic integers

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Let $a = (a_1, a_2, a_3, \ldots)$ be a sequence of positive integers. Define the operator on the ring of the formal power series with the integer coefficients, by the following rule: $A(f_0 + f_1x + f_2x^2 + \cdots) = a_1f_1 + a_2f_2x + a_3f_3x^2 + \cdots$. We consider the equation

$$(Ay)(x) + f(x) = y(x),$$

where $f(x) = f_0 + f_1x + f_2x^2 + \cdots \in \mathbb{Z}[[x]]$. We study solutions of this equation in $\mathbb{Z}[[x]]$.

Using a-adic integers, we prove that the equation has the following unique solution in $\mathbb{Z}_a[[x]]$:

$$y(x) = f(x) + (Af)(x) + (A^2f)(x) + (A^3f)(x) + \cdots .$$

We can prove that this solution belongs to $\mathbb{Z}[[x]]$ if and only if the sum of the series $f_0 + a_1f_1 + a_1a_2f_2 + a_1a_2a_3f_3 + a_1a_2a_3a_4f_4 + \cdots$ is integer in the ring $\mathbb{Z}_a$.

Notice, that if $a = (1, 2, 3, 4, \ldots)$, then A is the differentiation operator. Then the equation is written as $y'(x) + f(x) = y(x)$ and if $a = (b, b, b, b, \ldots)$, then $A = b \cdot S^*$, where S^* is a backward shift operator. Then the equation is written as $bS^*(y)(x) + f(x) = y(x)$ and for the integer coefficients y_n of the series y we have the difference equation $by_{n+1} + f_n = y_n$. Moreover, the solution of this system obtained with the aid of some analog of Cramer's rule is the only solution we need in $\mathbb{Z}[[x]]$.

On feebly compact semitopological symmetric inverse semigroups of a bounded finite rank

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Given a positive integer number n, we discuss feebly compact T_1-topologies τ on the symmetric inverse semigroup $\mathscr{I}_\lambda^n$ of finite transformations of rank $\leqslant n$ such that $(\mathscr{I}_\lambda^n, \tau)$ is a semitopological semigroup.

Paper reference: (**http://arxiv.org/abs/1708.02064**).

A new metric on the hyperspace of finite sets of a metric space

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Let (X, d) be a metric space. For every $n \in \mathbb{N}$ and $p \in [1, \infty]$ we endow the power X^n with the ℓ_p-metric $d^p_{X^n}$ defined by

$$d^p_{X^n}\big((x_i)_{i\in n}, (y_i)_{i\in n}\big) = \begin{cases} \sqrt[p]{\sum_{i\in n} d_X(x_i, y_i)^p} & \text{if } p < \infty; \\ \max_{i\in n} d_X(x_i, y_i) & \text{if } p = \infty. \end{cases}$$

On the set FX of non-empty finite subsets of X consider the largest metric d^p_{FX} such that for every $n \in \mathbb{N}$ the map

$$X^n \to FX, \ \ (x_1, \dots, x_n) \mapsto \{x_1, \dots, x_n\}$$

is non-expanding.

We prove that the metric d^∞_{FX} coincides with the well-known Hausdorff metric on FX. We are interested in the metrics d^p_{FX} on FX for $p < \infty$.

The distance d^1_{FX} can have applications in economics as it represent the cost of transportation of goods whose mass is negligible comparing to the mass of the transporting car.

For the metric d^1_{FX} many natural problems appear. In particular, the problem of finding an (efficient) algorithm for calculation of the distance between two finite subsets of $\mathbb{R}^n$, see (**https://mathoverflow.net/questions/277604/a-new-ell-p-metric-on-the-hyperspace-of-finite-sets**).

Some analogies between Haar meager sets and Haar null sets

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In 2013 Darji [2] defined a σ-ideal of "small" sets in an abelian Polish group which is equivalent to the family of meager sets in a locally compact group. He was motivated by Christensen's paper [1] where the author defined Haar null set in an abelian Polish group in such a way that in a locally compact group it is equivalent to the notion of Haar measure zero set.

We present interesting properties of Haar meager sets, especially those which are analogous to properties of Haar null sets.

1. J. P. R. Christensen, *On sets of Haar measure zero in abelian Polish groups*, Israel J. Math. **13** (1972), 255–260.
2. U. B. Darji, *On Haar meager sets*, Topology Appl. **160** (2013), 2396–2400.

Homotopic Baire classes

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We introduce homotopic Baire classes. Namely, a map $f : X \to Y$ between topological spaces belongs to the first (homotopic) Baire class if there exists a sequence of continuous map $f_n : X \to Y$ such that (f_n is homotopic to f_k for all $n, k \in \mathbb{N}$ and) $\lim_{n\to\infty} f_n(x) = f(x)$ for every $x \in X$. The collection of all maps of the first (homotopic) Baire class between X and Y is denoted by $B_1(X, Y)$ ($hB_1(X, Y)$).

Theorem. *Let X be a topological space and Y be a metrizable connected ANR. Then $B_1(X, Y) = hB_1(X, Y)$.*

On asymptotic sublogarithmic dimension

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The asymptotic dimension of a (proper) metric space is defined by M. Gromov [1] and is one of the most important invariants in large scale geometry.

Some other asymptotic dimension functions were also considered: the asymptotic Assouad-Nagata dimension [2], asymptotic dimension with linear control [3], and asymptotic power dimension [1].

The aim of the talk is to introduce a new asymptotic dimension function, namely, the asymptotic sublogarithmic dimension. We establish some basic properties of this dimension and compare it to another dimension functions.

We also introduce the sublogarithmic coarse structure and the sublogarithmic corona of a proper metric space.

1. M. Gromov, *Asymptotic invariants of infinite groups*, Geometric Group Theory, Vol. **2**, Cambridge University Press, 1993.
2. A. N. Dranishnikov and J. Smith, *On asymptotic Assouad-Nagata dimension*, Topology Appl. **154**:4 (2007), 934–952.
3. C. Bedwell, *On asymptotic dimension with linear control*, arXiv:1404.0421v1, 2014
4. J. Kucab and M. Zarichnyi, *On asymptotic power dimension*, Topology Appl. **201** (2016), 124–130.

Homotopy properties of Morse functions on the Möbius strip

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For any manifold X denote by $D(X,Y)$ the group of diffeomorphisms of X fixed on a closed subset $Y \subset X$. For each $f \in C^{\infty}(X,\mathbb{R})$ define the stabilizer and the orbit with respect to the right action of the group $\mathcal{D}(X,Y)$ on the space $C^{\infty}(X,\mathbb{R})$ as follows:

$$\mathcal{S}(f,Y) = \{h \in \mathcal{D}(X,Y) \mid f \circ\ h = f\}, \quad \mathcal{O}(f,Y) = \{f \circ h \mid h \in \mathcal{D}(X,Y)\}.$$

and endow them with Whitney C^{∞}-topologies.

Let M be the Möbius strip, $\mathrm{Morse}(M,\mathbb{R})$ be the set of functions $f \in C^{\infty}(M,\mathbb{R})$ satisfying the following properties:

1) f is constant on the boundary ∂M,
2) all its critical points are non-degenerate and belong to the interior of M.

Let $\Gamma(f)$ be the Kronrod–Reeb graph being the factor space of M by the partition into connected components of level-sets $f^{-1}(c)$, $c \in \mathbb{R}$. It is well known that $\Gamma(f)$ has the structure of a graph for each $f \in \mathrm{Morse}(M, \mathbb{R})$. Moreover, $\Gamma(f)$ is a tree. Let also $p\colon M \to \Gamma(f)$ be the corresponding factor map to the Kronrod–Reeb graph. Notice that $\mathcal{S}(f)$ acts on the $\Gamma(f)$ by the rule $(h, x) \mapsto p \circ h \circ p^{-1}(x)$, where $h \in \mathcal{S}(f)$, $x \in \Gamma(f)$.

Theorem. *Let $f \in \mathrm{Morse}(M, \mathbb{R})$. Then the following statements hold true.*

1) There is a vertex u of $\Gamma(f)$ fixed with respect to the action of $\mathcal{S}(f)$ and $\mathcal{S}(f)$-invariant regular neighborhood U of u such that the connected components of $\overline{M \setminus p^{-1}(U)}$, enumerated by $X_0, X_1, \ldots, X_n$, have the following properties:
   - $\partial M \subset X_0$, X_0 is diffeomorphic to $S^1 \times [0, 1]$,
   - X_i is diffemorphic to a 2-disk for all $i \leq n$.
2) Suppose for all $h \in \mathcal{S}(f, \partial M)$ and for each $i \in \{1, 2, \ldots, n\}$ we have $h(X_i) = X_i$ and h preserves orientation of X_i. Then there are the following isomorphisms: $\pi_1\mathcal{O}(f, \partial M) \simeq \pi_0\mathcal{S}(f, \partial M) \simeq \prod_{i=1}^{n} \pi_0\mathcal{S}(f|_{X_i}, \partial X_i) \times \mathbb{Z}$.

The monad of free restricted Lie algebras

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We use elementary theory of monads in order to describe free restricted Lie algebras. All algebras and vector spaces are over a field k of characteristic $p \geq 0$. For a vector space X (resp. a Lie algebra g) the tensor algebra TX (resp. the universal enveloping algebra Ug) becomes a bialgebra, when equipped with the comultiplication $\Delta x = x \otimes 1 + 1 \otimes x$ for $x \in X$ (resp. $x \in g$). The subspace of primitive elements $\mathrm{Prim}B$ of a bialgebra B is defined as $\mathrm{Prim}B = \{y \in B \mid \Delta y = y \otimes 1 + 1 \otimes y\}$. The monad L in the category of vector spaces is defined as $LX = \mathrm{Prim}TX$. When characteristic p of field k is 0, it is known that LX coincides with the free Lie algebra $\mathrm{Li}X \subset TX$ generated by X. When the characteristic $p > 0$, $LX \supset \mathrm{Li}X$ is the free restricted Lie algebra generated by X. For any Lie algebra g we have $g = \mathrm{Prim}Ug$ (for $p \geq 0$) and for any restricted Lie algebra g we have $g = \mathrm{Prim}(ug)$ (for $p > 0$), where $ug = Ug/(x^p - x^{[p]})_{x \in g}$ is the restricted universal enveloping algebra of

g. Notice that $U(\mathrm{Li}X)$ is isomorphic to TX (for $p \geq 0$) and to $u(LX)$ (for $p > 0$). Using the Poincare-Birkhoff-Witt theorem one proves that a basis of LX (for $p > 0$) can be taken as $\{h^{p^n} \mid n \geq 0, h \in H\}$, where H is a basis of $\mathrm{Li}X \subset TX$.

On the Shafarevich-Tate group of elliptic curves over the rationals

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We present an overview of the Shafarevich-Tate and Selmer groups of an elliptic curve in the framework of Galois cohomology.

Positive series whose partial sumsets are Cantorvals

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The *achievement set* (or the *partial sumset*) of a series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \cdots + a_n + \cdots = S_n + (a_{n+1} + a_{n+2} + \cdots) = S_n + r_n$$

is the set $E\{a_n\} \equiv \left\{x : \ x = x(M) = \sum_{n \in M \subset \mathbb{N}} a_n, \quad M \in 2^{\mathbb{N}}\right\}$. Each element of $E\{a_n\}$ is called an *incomplete sum* of the series.

A *symmetric Cantorval* (for short a *Cantorval*) is a subset of $\mathbb{R}$ homeomorphic to

$$C_0 \cup \bigcup_{n=1}^{\infty} G_{2n-1} = [0,1] \setminus \bigcup_{n=1}^{\infty} G_{2n},$$

where G_k denotes the union of open middle thirds which are removed from $[0,1]$ at the k-th step in the construction of the Cantor ternary set C_0. This set is a union of nowhere dense set and a set, which is an infinite union of intervals.

More precisely, a *symmetric Cantorval* ($\mathcal{M}$-*Cantorval*, see [3]) is a non-empty compact subset $S \subset \mathbb{R}^1$ such that:

- S is the closure of its interior;
- both endpoints of any non-degenerated component of S are accumulation points of one-point components of S.

For each $\varepsilon > 0$ we present a convergent series $\sum_{n=1}^{\infty} a_n = 1$ with positive terms for which the following properties holds:

1) the sequence of ratios (a_n/r_n) of series terms to the corresponding series remainders is unbounded;
2) the achievement set (partial sumset) of the series is a symmetric Cantorval and its Lebesgue measure satisfies the following inequality: $0 < \lambda([0;1] \setminus E\{a_n\}) < \varepsilon$.

1. T. Banakh, A. Bartoszewitcz, M. Filipczak, and E. Szymonik, *Topological and measure properties of some self-similar sets*, Topol. Methods Nonlinear Anal. **46**:2 (2015), 1013–1028.
2. J. A. Guthrie and J. E. Nymann, *The topological structure of the set of subsums of an infinite series*, Colloq. Math. **55**:2 (1988), 323–327.
3. P. Mendes and F. Oliveira, *On the topological structure of the arithmetic sum of two Cantor sets*, Nonlinearity **7** (1994), 329–343.

S-dimension is equal to Hölder dimension in Peano continua

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In this talk, we resolve an old problem (2007, [1]) proving the equality of S-dimension and Hölder dimension in Peano continua.

Let X be a locally connected compact metrizable space. By $\mathcal{M}(X)$ we denote the family of metrics generating the topology of X. For any metric $d \in \mathcal{M}(X)$ and $\varepsilon > 0$ denote by $S_\varepsilon(X, d)$ the smallest cardinality of a cover of X by connected subsets of d-diameter $< \varepsilon$.

The value (finite or infinite) $S\text{-Dim}(X, d) = \limsup_{\varepsilon\to 0} \frac{\ln S_\varepsilon(X,d)}{\ln(1/\varepsilon)}$ is called *S-dimension*, its minimum $S\text{-dim}(X) = \inf_{d\in\mathcal{M}(X)} S\text{-Dim}(X, d)$ is called *the topological S-dimension* of X.

Next, we recall the definition of the Hölder dimension of a Peano continuum, i.e., a continuous image of the unit interval $[0, 1]$. By Hahn-Mazurkiewicz Theorem (1914), a compact metrizable space is a Peano continuum if and only if it is connected and locally connected.

A function $f : X \to Y$ between two metric spaces (X, d_X) and (Y, d_Y) is called α-*Hölder* if there exists a positive real constant L such that

$$d_Y(f(x), f(x')) \le L \cdot d_X(x, x')^\alpha$$

for all $x, x' \in X$. For a Peano continuum X its *topological Hölder dimension* is defined as

$$H\ddot{o}\text{-dim}(X) = \inf_{d \in \mathcal{M}(X)} H\ddot{o}\text{-Dim}(X, d)$$

where

$$H\ddot{o}\text{-Dim}(X, d) = \inf\left\{\alpha \in (0, \infty] : \exists \text{ an } \tfrac{1}{\alpha}\text{-Hölder map } f : [0, 1] \to X\right\}$$

is the *Hölder dimension* of the metric Peano continuum (X, d).

Theorem. *Any metric Peano continuum* (X, d) *has*

$$S\text{-Dim}(X, d) = H\ddot{o}\text{-Dim}(X, d) \quad \textit{and} \quad S\text{-dim}(X) = H\ddot{o}\text{-dim}(X).$$

1. T. Banakh and M. Tuncali, *Controlled Hahn-Mazurkiewicz Theorem and some new dimension functions of Peano continua*, Topology Appl. **154**:7 (2007), 1286–1297.

Topological stability of the averages of functions

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In the paper [1] we considered the problem of topological stability of a continuous function with respect to averaging by a discrete measure; in [2] this problem was studied for averaging by a measure having a locally constant density.

In this talk I will discuss the problem of topological stability of continuous functions with respect to averaging by a probability measure which is a convex combination of a discrete measure and a measure with locally constant density.

1. S. Maksymenko and O. Marunkevych, *Topological stability of functions with respect to averagings*, Ukr. Math. J. **68**:5 (2016), 642–656.
2. S. Maksymenko and O. Marunkevych, *Topological stability of continuous functions with respect to averaging by measures with locally constant densities*, Proc. of Intsitute of Math. of Ukrainian NAS **8**:6 (2015), 146–163.

The cluster sets of quasi-locally stable functions

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The notion of a cluster set was first formulated by P. Painleve in the study of analytic functions of a complex variable. A fundamental research of cluster sets of analytic functions was carried out in the monograph of E. F. Collingwood and A. J. Lohwater. Actually, the notion of cluster sets is a topological notion which characterizes a behavior of a function on the boundary of the domain of this function. The oscillation is another limit characteristic of functions which is tightly connected with the notion of cluster sets. Namely, the oscillation is the diameter of the cluster set.

The problem of the construction of functions with the given oscillation was investigated in numerous articles of P. Kostyrko, J. Ewert, S. Ponomarev, Z. Duszynski, Z. Grande. S. Kowalczyk and O. Maslyuchenko. In our previous papers we studied the question on the limit oscillation of locally stable functions and continuous functions, respectively. Now we investigate the question about the construction of functions with the given cluster sets.

Let X and Y be topological spaces, $D \subseteq X$ and $f : D \to Y$. The cluster set of f at a point $x \in X$ is defined by $\overline{f}(x) = \bigcap_{U \in \mathcal{U}_x} \overline{f(U \cap D)}$, where $\mathcal{U}_x$ denotes the collection of all neighborhoods of x in X. A subset A of X is called

- *quasi-open*, if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$;
- *quasi-closed*, if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$;
- *quasi-clopen*, if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A \subseteq \mathrm{cl}(\mathrm{int}(A))$.

A function $f : X \to Y$ is called *quasi-locally stable* if for any $x \in X$ there is a quasi-clopen subset A of X such that $x \in A$ and f is stable on A.

Proposition. *Let X, Y be topological spaces, $f : X \to Y$ and Y_d be a space Y, endowed with the discrete topology $\tau_d = 2^Y$. Then the following conditions are equivalent:*

- *f is quasi-locally stable;*
- *for any $y \in Y$ the preimage $f^{-1}(y)$ is a quasi-open set;*
- *for any $y \in Y$ the preimage $f^{-1}(y)$ is a quasi-clopen set;*
- *$f : X \to Y_d$ is quasi-continuous.*

In particular, every quasi-locally stable function is quasi-continuous.

Theorem. *Let X be a metrizable topological space, Y be a dense subspace of a metrizable compact space $\overline{Y}$, L be a closed nowhere dense subset of X, $\Phi : L \multimap \overline{Y}$ be an upper continuous compact-valued multifunction and $D \subseteq X \setminus L$*

such $L \subseteq \overline{D}$. Then there exists a quasi-locally stable function $f : D \to Y$ such that $\overline{f}(x) = \Phi(x)$ for any $x \in L$.

Separately continuous mappings with non-metrizable range

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Separately continuous mappings have been intensively studying during last 25 years. In particular, mappings with values in inductive limits, strongly σ-metrizable and σ-metrizable spaces, Moore spaces, stratifiable and semi-stratifiable spaces, the Sorgenfrey line, the Nemytsky, Bing and Ceder planes have been considered, see [1, 2, 3, 4]. The first part of the report will be devoted to the analysis of the results of these works.

At the same time, there are still many intriguing problems in this branch, so it is still far from its completion. Some of the mentioned problems deal with the Bing plane $\mathbb{B}$ [5, p. 518], which is an example of countable Hausdorff non-regular connected first-countable space, well-known Sorgenfrey line $\mathbb{L}$ [5, p. 47] and the rational line $\mathbb{Q}$ (see [6, 7]).

Problem 1. Describe the sets $D(f)$ of discontinuity points of the following separately continuous mappings:

(i) $f : \mathbb{L} \times \mathbb{Q} \to \mathbb{L}$;
(ii) $f : \mathbb{L}^2 \to \mathbb{B}$.

The following two theorems give partial answers to Problem 1(ii).

Theorem 1. *The set $L_c(f)$ of local constancy points of a separately continuous mapping $f : \mathbb{L}^2 \to \mathbb{B}$ is open and everywhere dense in $\mathbb{L}^2$ and for each point $b \in \mathbb{L}$ the set $D_b(g) = \{x \in \mathbb{L} : (x, b) \in D(f)\}$ of discontinuity points of the restriction $g = f|_E$, where $E = L_c(f) \cup (\mathbb{L} \times \{b\})$, is meager in $\mathbb{L}$.*

Theorem 2. *For any $b \in \mathbb{L}$ there exists a separately continuous function $f : \mathbb{L}^2 \to \mathbb{B}$, such that $D_b(f) = \mathbb{L}$.*

Both results can be generalized, and it will be discussed in the second part of the report.

Theorem 2 yields an example of a σ-metrizable space $Z = \mathbb{B}$, first-countable Baire spaces $X = Y = \mathbb{L}$ such that the product $X \times Y$ is Baire and a separately continuous mapping $f : X \times Y \to Z$, which has no continuity points at the horizontal line $X \times \{b\}$. We note that the set $C(f)$ of continuity points of such mapping is always residual, hence everywhere dense in the product $X \times Y$. Another interesting example can be obtained due to the following construction.

Theorem 3. *Let $X = 2^\omega$ be the Cantor cube, $Y = [0, \omega]$ be the space of ordinals from 0 to ω, endowed with its natural order topology, $Z = 2^{\le\omega} = 2^\omega \cup 2^{<\omega}$ be the space of all infinite and finite binary sequences, endowed with the topology consisting of the sets W whose intersection $W \cap 2^\omega$ is open in the cube 2^ω and for any $z \in W \cap 2^\omega$ there exists a number N such that $z|n = z|_{[0,n)} \in W$ for all $n \ge N$. Then the mapping $f : X \times Y \to Z$, defined by the rule $f(x, y) = x|y$, is separately continuous and $D(f) = X \times \{\omega\}$.*

We remark that the space Z in this example is sequential, cosmic, regular, σ-compact and σ-metrizable.

In conclusion, we shall formulate one more question:

Problem 2. Do there exist non-empty topological spaces X, Y, such that Y is the first-countable, the product $X \times Y$ is Baire, $Z = \lim\operatorname{ind} Z_n$, the inductive limit of the sequence of locally convex spaces Z_n, such that, Z is σ-metrizable, and separately continuous mapping $f : X \times Y \to Z$ with $D_b(f) = X \times \{b\}$ for some $b \in Y$?

1. T. Banakh, *Quasicontinuous and separately continuous functions with values in Piotrowski and Maslyuchenko spaces*, Proc. Sci. Conf., dedicated to the 100th anniversary of K. M. Fishman and M. K. Fage, Chernivtsi (July, 1-4, 2015), p.122–127.
2. T. Banakh, *Quasicontinuous and separately continuous functions with values Maslyuchenko spaces*, preprint (https://arxiv.org/abs/1506.01661)
3. V. K. Maslyuchenko, O. D. Myronyk, and O. I. Filipchuk, *Joint continuity of separately continuous mappings with values in completely regular spaces*, (to appear).
4. B. Cascales, A. J. Guirao, V. Kadets, and M. Soloviova, *Γ-flatness and Bishop-Phelps-Bollobas type theorems for operators*, preprint (https://arxiv.org/pdf/1704.01768.pdf).
5. R. Engelking, *General Topology*, Mir, Moscow, 1986.
6. V. K. Maslyuchenko and O. D. Myronyk, *Discontinuity points of separately continuous mappings with values in the Sorgenfrey line*, Mat. Stud. **45** (2016), 67–75.
7. V. K. Maslyuchenko and O. I. Filipchuk, *Discontinuity points of at most countable valued separately continuous mappings*, (to appear).

Invariant idempotent measures in topological spaces

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The notion of invariant (self-similar) measure for an iterated function system (IFS) of contractions on a complete metric space is introduced in [1]. The existence of invariant measures is proved by using the Banach contraction principle for suitable metrization of the set of probability measures on a metric space. The invariant measures impose an additional structure on the invariant set for the given IFS.

The aim of the talk is to introduce the invariant idempotent measures for given IFS. Recall that an idempotent measure on a compact Hausdorff space X is a functional $\mu\colon C(X)\to\mathbb{R}$ that preserves constants, the maximum operation (usually denoted by $\oplus$) and is weakly additive (i.e., preserves sums in which at least one summand is a constant function; we use $\odot$ for these sums) [3]. Given an arbitrary metric space X, we denote by $I(X)$ the set of idempotent measures of compact support on X. In the case of idempotent measure, we use the weak* convergence for proving the existence of invariant element. This approach seems to be fairly general and we anticipate new results in this direction. Note also that the invariant idempotent measures on ultrametric spaces are introduced and investigated in [2].

Let X be a complete metric space and let $f_1, f_2, \dots, f_n$ be an IFS on X. We assume that all f_i are contractions. Let also $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$ be such that $\bigoplus_{i=1}^n \alpha_i = 0$.

By $\exp X$ we denote the hyperspace of the space X endowed with the Hausdorff metric. Let Ψ_0 denote the identity map of $\exp X$ and, for $i>0$, define $\Psi_i\colon \exp X \to \exp X$ inductively: $\Psi_i(A) = \bigcup_{j=1}^n f_j(\Psi_{i-1}(A))$.

Let $\Phi_0\colon I(X)\to I(X)$ be the identity map. For $i>0$, define $\Phi_i\colon I(X)\to I(X)$ inductively: $\Phi_i(\mu) = \bigoplus_{j=1}^n \alpha_j \odot I(f_j)(\Phi_{i-1}(\mu))$. Thus, $\Phi_i = \Phi_1\Phi_1\cdots\Phi_1$ (i times). It is easy to check that the maps Φ_i are well-defined.

In this case, we say that $\mu\in I(X)$ is an *invariant idempotent measure* if $\Phi_i(\mu)=\mu$ for every $i\in\mathbb{N}$ (equivalently, $\Phi_1(\mu)=\mu$).

Theorem. *There exists a unique invariant idempotent measure for the IFS $f_1, f_2, \dots, f_n$ and $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$ with $\bigoplus_{i=1}^n \alpha_i = 0$. This invariant measure is the limit of the sequence $(\Phi_i(\mu))_{i=1}^\infty$, for arbitrary $\mu\in I(X)$.*

1. J. E. Hutchinson, *Fractals and self-similaruty*, Indiana Univ. Math. J. **30** (1981), 713–747.
2. N. Mazurenko and M. Zarichnyi, *Idempotent ultrametric fractals*, Visnyk Lviv Univ. Series Mech. Math. **79** (2014), 111–118.
3. M. Zarichnyi, *Spaces and maps of idempotent measures*, Izvestiya: Mathematics, **74**:3 (2010), 481–499.

Banach representations of topological groups and dynamical systems

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We study representability of topological groups and dynamical systems on Banach spaces. In particular, on Rosenthal spaces. A Banach space is said to be *Rosenthal* if it does not contain an isomorphic copy of l_1.

Every circularly ordered dynamical system is representable on a Rosenthal space.

For every circularly ordered compact space the group of all circular order preserving homeomorphisms in the compact open topology is Rosenthal representable.

Sturmian like finite colorings of groups can be encoded as a matrix coefficient of Rosenthal representations.

We are going also to discuss some open questions.

On the intermediate separately continuous function

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We will call a pair (g, h) of two functions $g, h : X \to \mathbb{R}$ with $g(x) \le h(x)$ on X an *ordered pair* on X, and we will call a function $f : X \to \mathbb{R}$ an *intermediate* for ordered pair (g, h), if $g(x) \le f(x) \le h(x)$ on X. For a topological space X an ordered pair (g, h) on X is called a *Hahn's pair* [1], if the functions g and h are upper and lower semicontinuous, respectively.

H. Hahn [2] discovered, that each Hahn's pair on metric space X has a continuous intermediate function. Later H. Tong [3] and M. Katetov [4] proved, that for T_1-spaces the existence of a continuous intermediate function for each Hahn's pair on X is equivalent to the normality of space X.

There is a lot of modifications of this theorem, and in the last years there have appeared new results on the existence of intermediate functions from different functional classes, such as monotonous, piecewise linear or differentiable functions [1]. In connection with this we naturally got the problem of existence of intermediate separately continuous function.

For the map $f : X \times Y \to Z$ and a point $(x, y) \in X \times Y$ we put $f^x(y) = f(x, y) = f_y(x)$. For topological spaces X, Y and Z we note by $C(X)$, $C^u(X)$ and $C^l(X)$ the spaces of continuous and upper or lower respectively semicontinuous functions $f : X \to \mathbb{R}$, by $CC(X \times Y)$, $C^uC^u(X \times Y)$ and $C^lC^l(X \times Y)$ — spaces of separately continuous and upper or lower separately semicontinuous functions $f : X \times Y \to \mathbb{R}$, and by $C(X, Y)$ and $CC(X \times Y, Z)$ — spaces of continuous maps $f : X \to Y$ and separately continuous maps $f : X \times Y \to Z$ respectively.

For topological spaces X and Y we will call the ordered pair (g, h) of functions $g \in C^uC^u(X \times Y)$ and $h \in C^lC^l(X \times Y)$ a *separate Hahn's pair*.

Let's recall, that *plus-topology* $\mathcal{C}$ on product $X \times Y$ of two topological spaces consists of sets $O \subseteq X \times Y$ such, that for each point $p = (x, y) \in O$ there exist neighborhoods U of point x and V of point y in spaces X and Y respectively with property $(U \times \{y\}) \cup (\{x\} \times V) \subseteq O$ (see, for example, [5]).

Theorem 1. *Let X and Y be a T_1-spaces. Then each separate Hahn's pair (g, h) on product $X \times Y$ has an intermediate separately continuous function if and only if space $Q = (X \times Y, \mathcal{C})$ is normal.*

The conditions on the space Q that make it a normal space have not been investigated yet, but in [6] it was proved, that the space $(\mathbb{R}^2, \mathcal{C})$ is not regular. This and Theorem 1 imply:

Theorem 2. *There is a separate Hahn's pair (g, h) on $\mathbb{R}^2$, that doesn't have an intermediate continuous function.*

1. V. K. Maslyuchenko, O. V. Maslyuchenko, and V. S. Melnyk, *Existence of intermediate piecewise linear and infinitely differentiable functions*, Bukovinian Math. J. **4**:3-4 (2016), 93–100.
2. H. Hahn, *Uber halbstetige und unstetige Functionen*, Sitzungsberichte Akad. Wiss. Wien. Math. – naturwiss. Kl. Abt. IIa. **126** (1917), 91–110.
3. H. Tong, *Some characterizations of normal and perfectly normal spaces*, Bull. Amer. Math. Soc. **54** (1948), 65.
4. M. Katetov, *Correction to 'On real-valued functions in topological spaces'*, Fund. Math. **40** (1953), 203–205.
5. H. A. Voloshyn, V. K. Maslyuchenko, and O.V. Maslyuchenko, *Immersion of the space of separately continuous function into the product of Banach spaces and its barrelledness*, Math. Bull. Shevchenko Scien. Soc. **11** (2014), 36–50.
6. V. V. Mykhaylyuk, *Topology of separate continuity and one generalisation of Sierpinski theorem*, Math. Stud. **14**:2 (2000), 193–196.

On the semigroup of order isomorphisms of principal filters of a power of the integers

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Given a positive integer number n put $\mathscr{IPF}(\mathbb{N}^n)$ is the semigroup of all order isomorphisms between principal filters of the n-th power of the set of positive integers $\mathbb{N}$ with the product order. We study algebraic properties of the semigroup $\mathscr{IPF}(\mathbb{N}^n)$, its semigroup topologizations and closures in (semi)topological semigroups.

Namioka spaces and o-game

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A topological (Baire) space X is called *Namioka*, if for every compact space Y and every separately continuous function $f : X \times Y \to \mathbb{R}$ there exists a dense in X G_δ-set $A \subseteq X$ such that f is continuous at every point of $A \times Y$.

J.P.R. Christesen in [1] used the topological game method for investigations of Namioka spaces for the first time. Christesen's result was generalized by many mathematicians (see [2] and literature therein). The following problem naturally arises.

Problem. Characterize the class of (completely regular) Namioka spaces in terms of topological games.

We introduce the following topological game which we call by *o-game*. The players β and α choose alternatively open nonempty sets

$$U_0 \supseteq V_1 \supseteq U_1 \supseteq V_2 \supseteq U_2 \dots$$

in a topological space X. The player β wins if the set $\bigcap_{n=1}^{\infty} V_n$ does not have isolated points and is open in X.

The following theorem is closely connected with the example [3] of non-Namioka Baire space.

Theorem 1. *Let X be a topological space, $U_0 \subseteq X$ be an open nonempty set, α be a limit ordinal and $(\mathcal{G}_\xi : 0 \le \xi < \alpha)$ be a sequence of families $\mathcal{G}_\xi = (G_{\xi,i} : i \in I_\xi)$ of disjoint clopen in X sets $G_{\xi,i} \subseteq U_0$ which satisfies the following conditions:*

(1) every set $G_\xi = \bigsqcup_{i \in I_\xi} G_{\xi,i}$ is dense in U_0;
(2) for every $0 \le \xi < \eta < \alpha$ and $j \in I_\eta$ there exists $i \in I_\xi$ such that $G_{\xi,i} \supset G_{\eta,j}$;
(3) $G_\eta = \bigcap_{\xi<\eta} G_\xi$ for every limit ordinal $\eta < \alpha$;
(4) $\bigcap_{\xi<\alpha} G_\xi = \emptyset$.

Then there exist a Hausdorff compact space Y and a separately continuous function $f : X \times Y \to \mathbb{R}$ such that for every $x \in U_0$ there exists $y_x \in Y$ such that f is jointly discontinuous at (x, y_x).

Corollary. *Let X be a β-o-favorable regular Baire space, X_0 be the first move of β according to a winning strategy in o-game on X and $(A_\xi : 0 \le \xi < \omega_1)$ be a sequence of nowhere dense in X_0 sets such that $X_0 = \bigcup_{0 \le \xi < \omega_1} A_\xi$. Then X is not a Namioka space.*

Theorem 2. *Let X be a β-o-unfavorable GO-space. Then X is a Namioka space.*

Theorem 3. *Let X be a GO-space such that there exists a sequence $(A_\xi : 0 \leq \xi < \omega_1)$ of nowhere dense in X sets with $X = \bigcup_{0\leq\xi<\omega_1} A_\xi$. Then X is X is a Namioka space if and only if β-o-unfavorable.*

Question 1. Let X be a completely regular Namioka space. Must X be β-o-unfavorable?

Question 2. Let X be a β-o-unfavorable. Must X be Namioka?

Question 3. Let X be a Namioka GO-space. Must X be β-o-unfavorable?

1. J. P. R. Christesen, *Joint continuity of separately continuous functions*, Proc. Amer. Math. Soc. **82**:3 (1981), 455–461.
2. V. V. Mykhaylyuk, *Namioka spaces and topological games*, Bull. Austral. Math. Soc. **73** (2006), 263–272.
3. M. Talagrand, *Espaces de Baire et espaces de Namioka*, Ann. of. Math. **270**:2 (1985), 159–164.

Selective versions of countable chain condition in bitopological spaces

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In this talk we present the selective version of the c.c.c. property in bitopological spaces and give some relations with other selective properties.

Gamma ideal convergence of a sequence of functions

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Using theory of the sets of ideal cluster functions, we define a new type ideal convergence. We obtain some results for families and sequences of functions. We also give some examples for describe their structure and properties.

Topological and metric properties of continuous fractal functions related to various systems of encoding for numbers

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In the talk, we construct continuous functions with complicated local topological and metric structure, variational, integral, differentional properties: non-differentiable, nowhere monotonic and singular (monotonic, non-monotonic, nowhere monotonic) functions. Probability distribution functions of singular random variables, functions preserving some properties of representation of numbers in some system of encoding of numbers, probability distribution functions of random variables such that they are functions of random argument, probability distribution functions of convolutions of anomalously fractal distributions, functions performing atomic transfer of mass are among them. We study local and global topological, metric and fractal properties of such functions, namely: 1) their sets of instability; 2) level sets; 3) graphs; 4) sets of peculiarities; 5) ability to preserve or transform fractal Hausdorff–Besicovitch dimension etc.

Extensions of the isomorphisms of free paratopological groups

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Let X be a topological space. Consider the equivalence relation on X by putting $x \sim y$ if x and y are not separated by open sets. In each equivalence class we take an arbitrary point and form from these points the set X_1. Up to a homeomorphism the space X_1 does not depend on the chosed points, is T_0 space. We call it T_0-*derivative* of the space X.

Theorem 1. *Let X and Y be topological spaces, X_1 and Y_1 their T_0-derivatives, $|X/X_1| = |Y/Y_1|$, F be one of the following functors: free paratopological group F_p, free abelian paratopological group A_p, free homogeneous space H. Then every topological isomorphism $i\colon F(X_1) \to F(Y_1)$ can be extended to a topological isomorphism $j\colon F(X) \to F(Y)$.*

Theorem 2. *Let X and Y be a completely regular spaces, X_1 and Y_1 be their T_0-derivatives, $|X/X_1| = |Y/Y_1|$, F be one of the following functors: free topological group F, free abelian topological group A. Then every topological isomorphism $i\colon F(X_1) \to F(Y_1)$ can be extended to a topological isomorphism $j\colon F(X) \to F(Y)$.*

On the idempotent barycenter map

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The problem of the openness of the barycentre map of probability measures was investigated in [1], [2], [3], [4] and [5]. In particular, it is proved in [4] that the barycentre map for a compact convex set in a locally convex space is open iff the map $(x, y) \mapsto \frac{1}{2}(x + y)$ is open.

Zarichnyj defined in [6] the idempotent barycentre map for idempotent measures and posed the problem of characterization of the openness the idempotent barycentre map. We show that the openness of the idempotent barycenter map is equivalent to the openness of the map of Max-Plus convex combination. As corollary we obtain that the idempotent barycenter map is open for the spaces of idempotent measures.

1. L. Q. Eifler, *Openness of convex averaging*, Glasnik Mat. Ser. III, **32**:1 (1977), 67вЂ“-72.
2. V. V. Fedorchuk, *On a barycentric map of probability measures*, Vestn. Mosk. Univ, Ser. I, **1** (1992), 42–47.
3. V. V. Fedorchuk, *On barycentrically open bicompacta*, Sib. Math. J. **33** (1992), 1135–1139.
4. R. C. O'Brien, *On the openness of the barycentre map*, Math. Ann. **223** (1976), 207–212.
5. S. Papadopoulou, *On the geometry of stable compact convex sets*, Math. Ann. **229** (1977), 193–200.
6. M. Zarichnyi, *Spaces and mappings of idempotent measures*, Izv. Ross. Akad. Nauk Ser. Mat. **74** (2010), 45–64.

Paper reference: (**http://arxiv.org/abs/1706.06823**).

On continuous images of some selected subsets of the real line

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Alexander Osipov asked whether it is true that for any Bernstein set A on the real line there are countable many continuous real functions such that the union of their images of A is whole $\mathbb{R}$.

We give the positive answer on this question. We present the other results connected with this question. Among of them we show that every coanalytic set which is not Borel cannot be image of any Bernstein set by the arbitrary continuous function.

Distributions of values of fractal functions related to Q_2-representation of real numbers

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Consider numbers belonging to $\langle 0;1\rangle$. Let $\Delta^g_{a_1\ldots a_n\ldots}$ be a fixed encoding (representation) for such numbers with zero redundancy and alphabet $A_2 = \{0,1\}$, i.e., surjective mapping

$$A_2 \times A_2 \times \ldots \times A_2 \times \ldots \equiv L \xrightarrow{g} \langle 0;1\rangle.$$

In particular, it can be a classic binary representation.

Let φ be a finite function defined on $A_2 \times A_2$ and having values on A_2.

Let f_φ be a function defined by equality

$$f_\varphi(\Delta^g_{a_1\ldots a_n\ldots}) = \Delta^g_{\varphi(a_1,a_2)\varphi(a_2,a_3)\ldots\varphi(a_{n-1},a_n)\varphi(a_n,a_{n+1})\ldots}.$$

This function is well-defined because we use only one of two representations for the same number.

For given g-representation of numbers belonging to $\langle 0;1\rangle$, the class $\mathfrak{F}$ of functions f_φ contains 16 functions [1]. Two of them are degenerated, one is trivial (identity transformation of $\langle 0;1\rangle$). Only four of them are continuous, namely above mentioned functions and inversor of digits of g-representation, i.e., function f_φ generated by mapping

$$\varphi(i,j) = 1 - i \quad \text{for all} \quad (i,j) \in A_2 \times A_2.$$

Remark that the class $\mathfrak{F}$ contains the left shift operator determined by mapping $\varphi(i,j) = j$ for any i and j from A_2. It is a piecewise linear function. Superposition of the shift operator and inversor is its "twin".

Theorem 1. *If $\varphi(i,j) = ij$, then the set of values of function f_φ is a nowhere dense set of zero Lebesgue measure and fractional Hausdorff–Besicovitch dimension.*

Let X be a random variable such that digits of its g-representation are independent random variables with a given distribution. We consider a distribution of random variable $Y = f_\varphi(X)$.

In the talk, we study structural (content of discrete and continuous components as well as singular and absolutely continuous components if continuous component is nontrivial), topological, metric and fractal properties of the spectrum of distribution of random variable Y.

In the sequel, we consider case when g-representation is a Q_2-representation of numbers and discuss some nontrivial facts related to this case.

Let q_0 be a fixed number belonging to $(0;1)$, let $q_1 \equiv 1 - q_0$, $\beta_0 = 0$, and $\beta_1 = q_0$ ($\beta_i = iq_{1-i}$). It is known [2] that for any $x \in [0;1]$ there exists a sequence $(\alpha_n) \in L$ such that

$$x = \beta_{\alpha_1} + \sum_{k=2}^{\infty}\left(\beta_{\alpha_k}\prod_{j=1}^{k-1} q_{\alpha_j}\right) = \Delta^{Q_2}_{\alpha_1\alpha_2\ldots\alpha_k\ldots}.$$

This equality determines an encoding of number x by means of alphabet A_2. It is called a Q_2-representation of numbers. Q_2-representation is a classic binary representation if $q_0 = \frac{1}{2}$.

Theorem 2. *If X has an uniform distribution and φ is one of the functions*

$$\varphi_3(i,j)=\begin{cases} 0 \text{ if } i(i+j) = 0, \\ 1 \text{ if } i(i+j) \neq 0; \end{cases} \quad \varphi_5(i,j)=\begin{cases} 0 \text{ if } j(i+j) = 0, \\ 1 \text{ if } j(i+j) \neq 0; \end{cases} \quad \varphi_6(i,j)=|i-j|,$$

then $Y = f_\varphi(X)$ is also an uniformly distributed random variable.

Theorem 3. *Let $\varphi \in \{\varphi_5, \varphi_6\}$. Random variable $Y = f_\varphi(X)$ and uniformly distributed random variable X have the same continuous distributions if and only if both digits of alphabet have the same probabilities.*

Theorem 4. *If X is a continuous random variable with independent identically distributed Q_2-digits and φ is one of the functions*

$$\varphi_1(i,j) = i \cdot j, \quad \varphi_7(i,j) = \begin{cases} 0, \text{ if } i+j = 0, \\ 1, \text{ if } i+j \neq 0; \end{cases}$$

then random variable $Y = f_\varphi(X)$ has a singularly continuous distribution of Cantor type.

Theorem 5. *If $X = \Delta^{Q_2}_{\tau_1\tau_2\ldots\tau_n\ldots}$ is a continuous random variable with independent identically distributed Q_2-digits, namely $P\{\tau_n = i\} = p_i$, $i = 0, 1$, and $\varphi \in \{\varphi_5, \varphi_6\}$, then 1) distribution of random variable Y is continuous if and only if distribution of random variable X is continuous; 2) random variables X and Y have singularly continuous distributions of Salem type if $\max\{p_0, p_1\} < q_0^{q_0} q_1^{q_1}$.*

Theorem 6. *If φ is one of the functions*

$$\varphi_2(i,j) = \begin{cases} 0 \text{ if } i(i+j) \neq 1, \\ 1 \text{ if } i(i+j) = 1; \end{cases} \qquad \varphi_4(i,j) = \begin{cases} 0 \text{ if } j(i+j) \neq 1, \\ 1 \text{ if } j(i+j) = 1, \end{cases}$$

and X is a continuous random variable with independent identically distributed Q_2-digits with nondegenerated distribution, then distribution of random variable $Y = f_\varphi(X)$ is singularly continuous of Cantor type.

We also consider metric spaces with specific metrics (in particular, metrics of Kullback type) as well as transformation of properties of functions using such metrics.

1. M. V. Pratsiovytyi and S. P. Ratushniak, *Distribution of values of one fractal function with random argument*, Trans. Natl. Pedagog. Mykhailo Drahomanov Univ. Ser. 1. Phys. Math. **16**:2 (2014), 150–160 (in Ukrainian).
2. M. V. Pratsiovytyi, *Random variables with independent Q_2-symbols*, Asymptotic methods in the study of stochastic models, Inst. Math. NAS Ukraine, Kyiv, 1987, 92–102 (in Russian).

Mapping class group for special singular foliations on a plane

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Let Δ be a 1-dimensional foliation on a plane with saddle-like singularites. This means there exists a finite discrete subset $\Sigma \subset \mathbb{R}^2$ such that the restriction of Δ on $\mathbb{R}^2 \setminus \Sigma$ is an usual 1-dimensional foliation, and for each point $q \in \Sigma$ there exist $n \geqslant 2$ and a neighbourhood U_q such that the restriction of Δ onto U_q is equivalent to the foliation by level-sets of the functions $\operatorname{Re} z^n$. Examples of such foliations are given by level-sets of pseudoharmonic function $f : \mathbb{R}^2 \to \mathbb{R}$ with finite number of saddle points.

Let T be the union of leaves whose closures contain singularities that intersect Σ. Then T can be regarded as the union of "topological" trees.

A homeomorphism $h : \mathbb{R}^2 \to \mathbb{R}^2$ is called Δ-*foliated* if $h(\omega) \in \Delta$ for each $\omega \in \Delta$. Let $H(\Delta)$ be the group of Δ-foliated homeomorphisms of the plane endowed with compact-open topology and $H_0(\Delta)$ be the identity path component of $H(\Delta)$. Then the quotient $\pi_0 H(\Delta) = H(\Delta) \diagup H_0(\Delta)$ is an analogue of the mapping class group for Δ-foliated homeomorphisms.

Let $H_{\mathbb{R}^2}(T)$ be the group of homeomorphisms of T which extend to homeomorphisms of the whole plane, and $\pi_0 H_{\mathbb{R}^2}(T)$ be the group of path components of $H_{\mathbb{R}^2}(T)$ with respect to compact-open topology.

Theorem. *If T is connected, then the groups $\pi_0 H(\Delta)$ and $\pi_0 H_{\mathbb{R}^2}(T)$ are isomorphic.*

A fixed point theorem for mappings on the ℓ_∞-sum of a metric space and its application

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The aim of this talk is to present a counterpart of the Banach fixed point principle for mappings $f : \mathbb{X}_\infty \to X$, where X is a metric space and $\mathbb{X}_\infty$ is the space of all bounded sequences of elements from X. Our result generalizes the theorem obtained by Miculescu and Mihail in 2008, who proved a counterpart of the Banach principle for mappings $f : X^m \to X$, where X^m is the Cartesian product of m copies of X. I will also compare our result with a recent one due to Secelean, who obtained a weaker assertion under less restrictive assumptions.

1. J. Jachymski, Ł. Maślanka, and F. Strobin, *A fixed point theorem for mappings on the ℓ_∞-sum of a metric space and its application*, Filomat, **31**:11 (2017), 3559-3572.

Criteria for product preservation of uniform continuity for a certain class of uniform spaces

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We give several criteria for the space $U(X)$ of uniformly continuous real-valued functions on metrizable uniform spaces X to be a ring. Some of these criteria apply to more general uniform spaces.

A new class of contractions in b-metric spaces and applications

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A class of b-contractions with respect to g, is introduced in the setting of b-metric spaces and existence and uniqueness of common fixed points for such mappings are established. Also, to support our results some examples and an application is provided for an integral equation.

Takahashi convexity structures in q-hyperconvex spaces

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In the context of this study, we investigated the convexity structures in the sense of Takahashi for T_0-quasimetric spaces, in an asymmetric setting. Especially, in the quasimetric theory similarly to the metric setting, some Takahashi convexity structures satisfying various interesting additional conditions are observed and studied in order to obtain strong results.

According to that, it is proved these generalized Takahashi convexity structures described in T_0-quasimetric spaces naturally occur in asymmetrically normed real vector spaces.

More specifically, in the final part we disclose that each q-hyperconvex T_0-quasimetric space can be endowed with a well-behaved convexity structure.

Max-min measures on compact Hausdorff spaces

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Let X be a compact Hausdorff space. A functional $\mu\colon C(X) \to \mathbb{R}$ is called a *max-min measure* if the following are satisfied:

1) $\mu(c) = c$ for every $c \in \mathbb{R}$;
2) $\mu(\varphi \vee \psi) = \mu(\varphi) \vee \mu(\psi)$ for every $\varphi, \psi \in C(X)$;
3) $\mu(c \wedge \varphi) = c \wedge \mu(\varphi)$ for every $c \in \mathbb{R}$ and every $\varphi \in C(X)$.

We endow the set $J(X)$ of max-min measures with the weak* topology.

The talk is devoted to the functor J in the category of compact Hausdorff spaces. In particular, we show that this functor is isomorphic to the functor I of idempotent measures. However, it turns out that the monads naturally generated by the functors I and J are not isomorphic.

I-Luzin sets

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We consider a notion of I-Luzin set which generalizes the classical notion of Luzin set and Sierpiński set on Euclidean spaces. We show that there is a translation invariant σ-ideal I with Borel base for which I-Luzin set can be I-measurable. If we additionally assume that I has the Smital property (or its weaker version) then I-Luzin sets are I-nonmeasurable. We give some constructions of I-Luzin sets involving additive structure of $\mathbb{R}^n$. Moreover, we show that if $\mathfrak{c}$ is regular, L is a generalized Luzin set and S is a generalized Sierpiński set then the complex sum $L + S$ belongs to Marczewski ideal s_0.

Section: Applications of Functional Analysis

On convergence of nonlinear integral operators at Lebesgue points

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We investigated pointwise convergence of nonlinear integrals at Lebesque points of generated function. We show that the main results applicable to a wide class of nonlinear integral operators, which may be constructed by using well known positive kernels in approximation theory.

1. F. Altomare, M. Campiti, and P. P. Korovkin, *Type approximation theory and its applications*, Walter de Gruyter, Berlin, New York, 1994.
2. L. Angeloni and G. Vinti, *Convergence in variation and rate of approximation for nonlinear integral operators of convolution type*, Results Math. **49** (2006), 1–23.
3. C. Bardaro, J. Musielak, and G. Vinti, *Nonlinear integral operators and applications*, de Gruyter Ser. Nonlinear Anal.Appl. **9** (2003), xii+201 pp.
4. C. Bardaro, G. Vinti, and H.Karsli, *Nonlinear integral operators with homogeneous kernels: pointwise approximation theorems*, Appl. Anal. **90**:3-4 (2011), 463–474.
5. P. L. Butzer and R. J.Nessel, *Fourier analysis and approximation*, Academic Press, New York and London, (1971), 553 p.
6. A. D. Gadjiev, A. Aral, and I. A. Aliev, *On behaviour of the Riesz and generalized Riesz potentials as order tends to zero*, Math. Ineq. Appl. *10*:4 (2007), 875–888.
7. I. A. Aliiev, A. D. Gadjiev, and A. Aral, *On approximation properties of family of linear operators at critical value of parameter*, J. Approx. Theory **38** (2006), 242–253.
8. 8. H. Karsli, *On approximation properties of non-convolution type nonlinear integral operators*, Anal. Theory Appl. **26**:2 (2010), 140–152.
9. J. Musielak, *Approximation by nonlinear singular integral operators in generalized Orlicz spaces*, Comment. Math. Prace Mat. **31** (1991), 79–88.
10. L. Rempulska and A. Thiel, *Approximation of functions by certain nonlinear integral operators*, Lith. Math. J. *48*:4 (2008) 451–462.

11. T. Swiderski and F. Wachnicki, *Nonlinear singular integral depending on two parameters*, Comment. Math. Prace Mat. **11** (2000), 181–189.
12. E. M. Stein and G. Weiss, *Introduction to fourier analysis on euclidean spaces*, Princeton, New Jersey, 1971.

Some approximation results of Szasz type operators including special functions

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In approximation theory, Szasz type operators and their generalizations and also modifications have been studied intensively by many researchers.The present paper deals with a class of Szasz type operators including special functions and their approximation properties.

Differential equations with separated variables on time scales

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We show that the well known theory for classical ordinary differential equations with separated variables is not valid in case of equations on time scales. Namely, the uniqueness of solutions does not depend on the convergence of appropriate integrals.

The analysis and geometry of Hardy's inequality and applications

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In this talk we present advances that have been made over recent decades in areas of research featuring Hardy's inequality and related topics. The inequality and its extensions and refinements are not only of intrinsic interest but are indispensable tools in many areas of mathematics and mathematical physics.

We will also discuss Hardy-type inequalities involving magnetic fields and Hardy, Sobolev and Cwikel-Lieb-Rosenblum inequalities for Pauli and Dirac operators.

Linear parabolic equations in variable Hölder spaces

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In my talk I will present my last results in Schauder theory for parabolic equations in variable Hölder spaces. Schauder theory is a priori estimates for parabolic equations. I want to obtain existence and uniqueness of solutions for linear parabolic equations by these Schauder inequalities. A proof of Schauder estimates is rather complicated. It consists of many smaller tasks. I use different techniques to prove these inequalities in interior of the set and other to prove it near boundary of the set. Then, these estimates yield the global Schauder inequalities.

Generalized Gaussian processes with applications to noncommutative functional analysis (operator spaces)

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In my talk we will present the following subjects:

1. Fock spaces of Yang-Baxter type.
2. Hecke operators.
3. Positivity of T-symmetrizators.
4. Connections of Woronowicz-Pusz operators with monotone Fock spaces of Muraki-Lu.
5. Noncommutative Levy processes for generalized "ANYON" statistics.
6. Operator spaces generated by generalized Gaussian operators.

Paper reference: M. Bożejko, E. Lytvynov, and J. Wysoczanski, *Fock representations of Q-deformed commutation relations*, J. Math. Phys. **58**, 073501 (2017).

Approximation by analogue of Zygmund sums in Lebesgue spaces with variable exponent

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Let $p = p(x)$ be a 2π-periodic measurable and essentially bounded function and let $L^{p(\cdot)}$ be the space of measurable 2π-periodic functions f such that $\int_{-\pi}^{\pi} |f(x)|^{p(x)}\, dx < \infty$. If $\underline{p} := \operatorname{ess\,inf}_x |p(x)| > 1$ and $\bar{p} := \operatorname{ess\,sup}_x |p(x)| < \infty$, then $L^{p(\cdot)}$ are Banach spaces [1] with the norm, which can be given by the formula

$$\|f\|_{p(\cdot)} := \inf\Big\{\alpha > 0 : \int_{-\pi}^{\pi} |f(x)/\alpha|^{p(x)}\, dx \le 1\Big\}.$$

We study the value

$$\mathcal{E}(L^{\psi}_{\beta,p(\cdot)}; \hat{Z}_n)_{s(\cdot)} := \sup\Big\{\|f - \hat{Z}_n(f)\|_{s(\cdot)} : f \in L^{\psi}_{\beta,p(\cdot)}\Big\}$$

upper bounds of the deviations of analogues of Zygmund sums

$$\hat{Z}_n(f;x) := a_0(f)/2 + \sum_{k=1}^{\infty} \Big(1 - \psi(n)/\psi(k)\Big)(a_k(f)\cos\ kx + b_k(f)\sin\ kx),$$

on the classes $L^{\psi}_{\beta,p(\cdot)} := \{f \in L^{\psi}_{\beta} :\ f^{\psi}_{\beta} \in U_{p(\cdot)}\}$, where L^{ψ}_{β} is the set of $(\psi;\beta)$-differentiable functions [2] ($\psi(k)$ is an arbitrary function of a natural variable, $\beta \in \mathbb{R}$) and $U_{p(\cdot)} := \{\varphi \in L^{p(\cdot)} :\ \|\varphi\|_{p(\cdot)} \le 1\}$ is the unit ball of $L^{p(\cdot)}$.

Theorem 1. *Let $(\psi;\beta) \in \Upsilon_{0,n}$ and $p, s \in \mathcal{P}^{\gamma}$, $s(x) \le p(x)$, $x \in [0;2\pi]$. Then for all $n \in \mathbb{N}$ the inequality*

$$C_{p,s}\nu(n) \le \mathcal{E}(L^{\psi}_{\beta,p(\cdot)}; \hat{Z}_n)_{s(\cdot)} \le K_{p,s}\nu(n)$$

holds, where $\nu(n) = \sup_{k\ge n} |\psi(k)|$, $C_{p,s}$ and $K_{p,s}$ are some constants that do not depend on n.

1. I. I. Sharapudinov, *On the topology of the space $L^{p(x)}([0;1])$*, Math. Motes **26**:4 (1979), 613–632.
2. A. I. Stepanets, *Methods of Approximation Theory*, VSP, Leiden–Boston, 2005.

Large Fourier Quasicrystals

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A set $E \subset \mathbb{R}^d$ is discrete if $E \cap B$ is finite for every ball $B \subset \mathbb{R}^d$ and uniformly discrete if $\inf\{|x - x'| :\ x, x' \in E,\ x \ne x'\} > 0$. A complex measure ν is *slowly increasing* if its variation $|\nu|$ on the ball $B(r)$ of radius r with center at 0 has a polynomial growth as $r \to \infty$, and *translation bounded* if its variations on every ball of radius 1 are uniformly bounded. If a measure ν on $\mathbb{R}^d$ is translation bounded, then $|\nu|(B(r)) = O(r^d)$.

A complex measure $\mu = \sum_{\lambda\in\Lambda} a_\lambda \delta_\lambda$ with discrete support Λ on $\mathbb{R}^d$ is a *Fourier Quasicrystal* if μ and its extended Fourier transform $\hat{\mu} = \sum_{\gamma\in\Gamma} b_\gamma \delta_\gamma$ are slowly increasing measures, and spectrum Γ is an arbitrary countable set.

Theorem (N. Lev, A. Olevskii, 2016)**.** *If Λ, Γ and $\Lambda - \Lambda$ are uniformly discrete, then the support Λ is a subset of a finite union of translates of a **unique** full-rank lattice L.*

Theorem (A. Cordoba, 1989)**.** *If a Fourier quasicrystal $\mu = \sum_{j=1}^{N} a_j \sum_{\lambda\in\Lambda_j} \delta_\lambda$ has a uniformly discrete support $\Lambda = \bigcup_j \Lambda_j$, and the Fourier transform $\hat{\mu}$ is a translation bounded measure, then the sets Λ_j are finite unions of translates of **several** full-rank lattices.*

We say $\mu = \sum_{\lambda\in\Lambda} a_\lambda \delta_\lambda$ is a *Large Fourier Quasicrystal* if $\inf_{\lambda\in\Lambda} |a_\lambda| > 0$.

Theorem 1. *If μ_1, μ_2 are Large Fourier Quasicrystals with supports Λ_1, Λ_2 and the set $\Lambda_1 - \Lambda_2$ discrete, then Λ_1 and Λ_2 are finite unions of translates of a* ***unique*** *full-rank lattice L.*

In the case $\mu_1 = \mu_2$ we get an analog of Lev–Olevskii Theorem for arbitrary countable spectrum. Also, we prove some generalization of the above result to "large" distributions with discrete support.

Theorem 2. *If μ is a Large Fourier Quasicrystal with a uniformly discrete support Λ and $|\hat{\mu}|(B(r)) = O(r^d)$ as $r \to \infty$, then Λ is a finite union of translates of* ***several*** *full-rank lattices.*

The proofs of the last two theorems are based on theory of almost periodic measures and generalization of Wiener's Theorem on Fourier series to Large Fourier Quasicrystals.

Boundary value problem for singularly perturbed parabolic equation of the second order on graphs

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Let Γ be a compact graph in $\mathbb{R}^3$ with vertices $V = \{a, a_1, \dots, a_{n_a}, b, b_1, \dots, b_{n_b}\}$ and edges $\{\gamma_1, \dots, \gamma_{n_{ab}}\}$. Each edge of the set $\{\gamma_1^a, \dots, \gamma_{n_a}^a\}$ is incident with the vertex a and the boundary vertex a_i $(i = 1, \dots, n_a)$. Similarly, each of the edges $\{\gamma_1^b, \dots, \gamma_{n_b}^b\}$ is incident to the vertex b and the boundary vertex b_i $(i = 1, \dots, n_b)$. Therefore, the degrees of a and b are equal to $\mathbf{deg}\, a = n_a + n_{ab}$, $\mathbf{deg}\, b = n_b + n_{ab}$ respectively. Let $\mathcal{I}^{ab} = \{1, \dots, n_{ab}\}$ be the set of indices of all edges that connect the vertices a and b. We will also denote by $\mathcal{I}^a = \{1, \dots, n_a\}$ and $\mathcal{I}^b = \{1, \dots, n_b\}$ the set of indices for the edges that are incident with vertex a and vertex b respectively. We call the set $\partial\Gamma = \{a_1, \dots, a_{n_a}, b_1, \dots, b_{n_b}\}$ the boundary of Γ.

We consider the singularly perturbed boundary value problem for parabolic equation on graph Γ with edges with a small heat conduction:

$$\partial_t u - \varepsilon^2 \partial_x^2 u + q(x)u = f(x,t), \quad (x,t) \in \Gamma \times (0,T), \tag{1}$$

with initial condition:

$$u(x,0) = \varphi(x), \quad x \in \Gamma, \tag{2}$$

a boundary condition:

$$u(x,t) = \mu(t), \quad (x,t) \in \partial\Gamma \times (0,T), \tag{3}$$

a continuity condition at common vertices a and b:

$$\begin{aligned} &u_{\gamma_1^a}(a,t) = u_{\gamma_2^a}(a,t) = \cdots = u_{\gamma_{n_a}^a}(a,t), \quad t \in (0,T), \\ &u_{\gamma_1^b}(b,t) = u_{\gamma_2^b}(b,t) = \cdots = u_{\gamma_{n_b}^b}(b,t), \quad\ \ t \in (0,T), \end{aligned} \tag{4}$$

the Kirchhoff conditions at common vertices a and b:

$$\Big(\sum_{i\in\mathcal{I}^a} \partial_{\gamma_i^a} u + \sum_{k\in\mathcal{I}^{ab}} \partial_{\gamma_k} u\Big)(a,t) = 0 = \Big(\sum_{j\in\mathcal{I}^b} \partial_{\gamma_i^b} u + \sum_{k\in\mathcal{I}^{ab}} \partial_{\gamma_k} u\Big)(b,t), \quad t \in (0,T), \tag{5}$$

where $\varepsilon > 0$ is a small parameter; u_{γ_s} is the restriction of u to the edge γ_s; $\partial_{\gamma_s} u(a,\cdot)$ is the value of the derivative with respect to dimensional variable at the vertex a along the edge γ_s in the direction from this vertex.

Under the smoothness conditions of the data of the problem full asymptotic expansion of the solution (1)-(5) are constructed. Asymptotic correctness is proved.

Some remarks on operators of stochastic differentiation in the Levy white noise analysis

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Operators of stochastic differentiation, which are closely related with the extended Skorohod stochastic integral and with the Hida stochastic derivative, play an important role in the Gaussian white noise analysis [1]. In particular, these operators can be used in order to study some properties of the extended stochastic integral and of solutions of so-called normally ordered stochastic equations.

During recent years operators of stochastic differentiation were introduced and studied, in particular, on spaces of regular and nonregular test and generalized functions of the Lévy white noise analysis, in terms of Lytvynov's generalization of the chaotic representation property [2]. But, strictly speaking, the existing theory in the "regular case" [3] is incomplete without a one more class of operators of stochastic differentiation. In particular, the above mentioned operators are relevant in calculation of the commutator between the extended stochastic integral and the operator of stochastic differentiation. We introduce this new class of operators and study some of their properties. Our researches can be considered as a contribution to further development of the Lévy white noise analysis.

1. T. Hida, H. H. Kuo, J. Potthoff, and L. Streit, *White Noise. An Infinite Dimensional Calculus*, Kluwer, Dordrecht, 1993.
2. E. W. Lytvynov, *Orthogonal decompositions for Lévy processes with an application to the gamma, Pascal, and Meixner processes*, Infinite Dimensional Analysis, Quantum Probability and Related Topics, **6**:1 (2003), 73–102.
3. M. M. Dyriv and N. A. Kachanovsky, *On operators of stochastic differentiation on spaces of regular test and generalized functions of Lévy white noise analysis*, Carpathian Math. Publ. **6**:2 (2014), 212–229.

The Boltzmann-Grad asymptotic behavior of observables of hard sphere fluids

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We develop a new formalism for the description of the kinetic evolution of a hard sphere fluids in the Boltzmann-Grad scaling limit within the framework of marginal observables. For initial states specied by means of a one-particle distribution function the relations between the Boltzmann-Grad asymptotic behavior of a nonperturbative solution of the Cauchy problem of the dual BBGKY hierarchy for hard spheres and a solution of the Boltzmann kinetic equation are established. One of the advantages of the stated approach to the derivation of kinetic equations from underlying hard sphere dynamics consists in an opportunity to construct the Boltzmann-like kinetic equation with initial correlations, in particular, that can characterize the condensed states of hard spheres. Moreover, it gives to describe the process of the propagation of initial correlations in the Boltzmann-Grad scaling limit. Using suggested approach, we also derive the non-Markovian Enskog kinetic equation with initial correlations and construct the marginal functionals of states, describing the creation of all possible correlations of particles with hard sphere collisions in terms of a one-particle distribution function governed by the stated Enskog equation.

Paper references:
(**http://arxiv.org/abs/1308.1789**, **doi:10.3934/krm.2012.5.459**).

Strong summability of double Fourier series

Ushangi Goginava

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In the talk I will discuss about the a.e. exponential strong summability problem for the rectangular partial sums of double trigonometric Fourier series of the functions from $L \log L$.

On trajectory and global attractors for non-autonomous reaction-diffusion equations with Caratheodory's nonlinearity

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In a bounded domain with sufficiently smooth boundary we investigate the uniform long-time behavior of all globally defined weak solutions of a non-autonomous reaction-diffusion system with Caratheodory's nonlinearity. We assume the standard sign and polynomial growth conditions on interaction function. We obtain new topological properties of weak solutions for the considered problem, prove the existence of the uniform trajectory and global attractors for all globally defined weak solutions of the problem, and provide the sufficient conditions for the existence of the uniform trajectory attractor in the strongest topologies. We present the example which shows that additional assumptions (for the existence of the uniform trajectory attractor in the strongest topologies) are essential. As applications we may consider the FitzHugh-Nagumo system (signal transmission across axons), the complex Ginzburg-Landau equation (theory of superconductivity), the Lotka-Volterra system with diffusion (ecology models), the Belousov-Zhabotinsky system (chemical dynamics) and many other reaction-diffusion type systems whose dynamics are well studied in the autonomous case and in the nonautonomous case, when all coefficients are uniformly continuous on a time variable. Now provided results allow us to study these systems with Caratheodory's nonlinearities.

Paper reference: (**doi:10.1016/j.na.2013.12.004**).

Solvability conditions of nonlocal boundary value problem for the differential-operator equation with weak nonlinearity

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We consider a nonlocal boundary value problem for a differential-operator equation with weak nonlinearity

$$\sum_{|\hat{s}|\le n} a_{\hat{s}} A_1^{s_1}\dots A_p^{s_p}\partial_t^{s_0}u(t)=\varepsilon u(t),$$

$$\mu\partial_t^m u\big|_{t=0}-\partial_t^m u\big|_{t=T}=0,\qquad m=0,1,\dots,n-1,$$

where $\hat{s}=(s_0,s)$, $s=(s_1,\dots,s_p)\in\mathbb{Z}_+^p$, $|\hat{s}|=s_0+s_1+\dots+s_p$, $a_{\hat{s}}$, ε and μ — complex parameters ($a_{n,0}$=1, $\mu\ne 0$), $\partial_t=\partial/\partial t$, A_j, $j=1,\dots,p$, — linear operators that have a common spectral representation.

The conditions of the sobvability of this problem are establish by using of the Nash-Mozer iteration scheme, where the main difficulty it is to get estimates of interpolation type for the inverse linearized operators obtained at each step of the iteration.

The problem is ill-posed in the Hadamard sense and its solvability depends on the small denominators which arise in the construction of the solution, so we used the metric approach.

Trigonometric approximation in grand Lebesgue spaces

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The direct and inverse problems of approximation theory in the subspace of weighted generalized grand Lebesgue spaces of periodic functions with the weights satisfying Muckenhoupt's condition are investigated. Appropriate direct and inverse theorems are proved. Moreover, the approximation properties of the matrix transforms are studied. As a corollary some results on constructive characterization problems in the generalized Lipschitz classes are presented.

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Financial model with disorder moment and mean square optimal hedging

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The model of risky asset price evolution is considered, that at the random time moment there is change of distribution of price process. Some properties of model are investigated and optimal in mean square sense hedging strategy is obtained for European contingent claim.

On strong global attractor for 3D Navier-Stokes equations in an unbounded domain of channel type

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We consider the modified three-dimensional Navier-Stokes system that coincides with the unmodified one in the case of limited velocity gradients. The problem is considered in unbounded domain satisfying the Poincare inequality. Theorem on the existence and uniqueness of solutions of the Cauchy problem is obtained. We obtain the existence of a global attractor for the corresponding semigroup in the strong topology of the phase space, show the proximity of these attractors to the set of complete bounded trajectories of 3D Navier-Stokes system.

Paper reference: (**doi:10.1615/JAutomatInfScien.v47.i11.40**).

The estimation of the probability of bankruptcy in the case of large payments and the determination of the optimal insurance fee

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Let the distribution function $F(x)$, $x \in \mathbb{R}_+ = [0; \infty)$ satisfies the condition $F(x) < 1 \ \forall x \in \mathbb{R}_+$ and belongs to the so-called subexponential type [1, p. 189].

Denote by S the class subexponential distribution functions. Note that the class S is wide enough. In particular, this class includes such distributions as Log-normal distribution, Pareto distribution, Weibull distribution, Benktander type I and type II distributions. Asymptotics of the probability of bankruptcy for some of these distribution considered in [1-3].

Considering insurance fees we will assume that the number of insurance contracts N, which are in the insurance portfolio, is a random value. Each insurance contract with the number j corresponds to the value S_j that we call the insured amount. Let Y_j – the value of the suit for the j-th agreement. We will also assume that random variables X_j and S_j are independent. Note that this is the essence of the F-model [4, 248].

Suppose also that for each insurance contract, the insurance fee Z_j is determined $Z_j = zS_j$ where z is some constant for all insurance contracts (insurance rate). Note that in the considered model fees are random variables depending on S_j which distinguishes it from classical statements of a problem.

Let us assume that claims under insurance contracts are large. Let us look for the asymptotic behavior of the optimal insurance rate in the case of F-models for relative claims distributed by Log-normal distribution, Pareto distribution, Weibull distribution, Benktander type I and type II distributions.

1. N. M. Zinchenko, *Mathematical methods in risk theory: a tutorial*, Publ. Center “Kyiv University”, 2008.
2. A. Bilynskyi and O. Kinash, *The probability of bancruptcy in case of subexponencial distributions of payment*, Collections sci. labor SWorld. **21**:1 (2015), 95–100.
3. A. Bilynskyi and O. Kinash, *The asymptotic behavior of the probability of bankruptcy for large payments*, Math. Comput. Modelling. Series: Phys. Math. Sc. **14** (2016), 5–10 (in Ukrainian).
4. V.Y. Korolev, V.E. Bening, and S.Y. Shorgin, *Mathematical foundations of risk theory*, Moscow, Fizmatlit, 2011, 620 p.

Stochastic differentia inclusions and applications

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The lecture contains the general informations and the main notions dealing with stochastic differential inclusions and some their applications. As the natural example of applications of the theory of stochastic differential inclusions, the basic problem of the stochastic optimal control theory is presented. As a particular example of applications of stochastic differential inclusions, the optimal portfolio selection problem is considered.

Existence and stability of traveling waves in parabolic systems with small diffusion

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Consider the equation [1, 2]

$$\frac{\partial u}{\partial t} = i\omega_0 u + \varepsilon\Big[(\gamma + i\delta)\frac{\partial^2 u}{\partial x^2} + (\alpha + i\beta)u\Big] + (d_0 + ic_0)u^2\overline{u} \tag{1}$$

with periodic boundary condition

$$u(t, x + 2\pi) = u(t, x), \tag{2}$$

where ε is a small positive parameter.

Theorem. *If $\omega_0 > 0$, $\alpha > 0$, $\gamma > 0$, $d_0 < 0$ and the condition $\alpha > \gamma n^2$ is satisfied for some $n \in \mathbb{Z}$, then for some $\varepsilon_0 > 0$, $0 < \varepsilon < \varepsilon_0$, the periodic on t solutions*

$$u_n = u_n(t, x) = \sqrt{\varepsilon} r_n \exp(i(\chi_n(\varepsilon)t + nx)) + O(\varepsilon)$$

of the boundary value problem (1), (2) exists. Here $r_n = \sqrt{(\alpha - n^2\gamma)\,|d_0|^{-1}}$, $\chi_n(\varepsilon) = \omega_0 + \varepsilon\beta + \varepsilon c_0 r_n^2 - \varepsilon\delta n^2$, $n \in \mathbb{Z}$. These solutions are exponentially orbitally stable if and only if the conditions

$$(d_0 r_n^2 - \gamma k^2)^2(\gamma^2 k^2 + \delta^2 k^2 - 2\gamma d_0 r_n^2 - 4\gamma^2 n^2 - 2\delta c_0 r_n^2) > 4\gamma^2 n^2 (c_0 r_n^2 - \delta k^2)^2$$

for all $k \in \mathbb{Z}$, $k \neq 0$, are fulfilled.

Consider the Brusselator equations with small diffusion

$$\frac{\partial u_1}{\partial t} = A - (B+1)u_1 + u_1^2 u_2 + \varepsilon d_1 \frac{\partial^2 u_1}{\partial x^2},$$

$$\frac{\partial u_2}{\partial t} = Bu_1 - u_1^2 u_2 + \varepsilon d_2 \frac{\partial^2 u_2}{\partial x^2}$$

with periodic boundary conditions

$$u_1(t, x + 2\pi) = u_1(t, x), \quad u_2(t, x + 2\pi) = u_2(t, x),$$

where $B = 1 + A^2 + \varepsilon$, $d_1 > 0$, $d_2 > 0$, ε is a small positive parameter. If $A > 0$ and the condition $1 > (d_1 + d_2)n^2$ is satisfied for some $n \in \mathbb{Z}$, then for some $\varepsilon_0 > 0$, $0 < \varepsilon < \varepsilon_0$, the periodic on t solutions

$$u_1 + iu_2 = \sqrt{\varepsilon} r_n \exp(i(At + nx)) + O(\varepsilon)$$

of the Brusselator model exists. Here $r_n^2 = \dfrac{4A^2}{A^2+2}\left(1 - (d_1 + d_2)n^2\right)$, $n \in \mathbb{Z}$.

1. I. I. Klevchuk, *Existence of countably many cycles in hyperbolic systems of differential equations with transformed argument*, J. Math. Sci. **215**:3 (2016), 341–349.
2. I. I. Klevchuk, *Bifurcation of auto-oscillations for parabolic systems with delay in the argument and small diffusion*, Nonlinear Oscil. **19**:3 (2016), 390–398.

Decomposable sets and set-valued Ito's integral

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In the presentation we establish some useful properties of decomposable hulls for subsets in a Banach space of integrable functions and their applications in set-valued analysis. Due to presented properties the problem of a boundedness of set-valued Ito's integral is analyzed as well as its consequences.

Paper reference: (**http://dx.doi.org/10.1016/j.jmaa.2014.11.041**).

Order-convex selections of multifunctions and their applications

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Let X be a Banach space and $(Y, <)$ be a Banach lattice. One of valuable problems in set valued analysis concerns the existence of regular selections of set-valued functions acting from X into nonempty subsets of Y. Investigating nonlinear differential inclusions, continuous, Lipschitz, differentiable or bounded variation selections are considered most often.

In the talk we introduce the class of "upper separated" set-valued functions and investigate the problem of the existence of order-convex selections of F. First, we present necessary and sufficient conditions for the existence of such selections. Next we discuss the problem of the existence of Carathéodory-order-convex type selections and apply obtained results to investigation of the existence and properties of solutions of differential and stochastic inclusions, like stability or lower-upper bounds. In second part of the talk we will discuss the applicability of obtained selection results to some deterministic and stochastic optimal control problems. Some examples will be presented also.

1. M. Michta and J. Motyl, *Locally Lipschitz selections in Banach lattices*, Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods **71**:5-6,(2009), 2335-2342.
2. J. Motyl, *Carathéodory convex selections of set-valued functions in Banach lattices*, Topol. Methods Nonlinear Anal. **43**:1 (2014), 1–10.
3. J. Motyl, *Stochastic retarded inclusion with Carathéodory-upper separated multifunctions*, Set-Valued Var. Anal. **24** (2016), 191–205.
4. J. Motyl, *Carathéodory-convex selections of multifunctions and their applications*, J. Nonlinear Convex Anal. **18** (2017), 535–551.

Dynamics of weak solutions for second-order autonomous evolution equation with discontinuous nonlinearity

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The considered equation of hyperbolic type describes the dynamics for a class of Continuum Mechanics processes, in particular, the piezoelectric processes with multivalued "reaction-displacement" law. Discontinuous interaction function is represented as the difference of subdifferentials of convex functionals. This case is actual for automatic feedback control problems. We study the dynamics of weak solutions of the investigated problem in terms of the theory of trajectory and global attractors for multivalued semi-flows generated by all weak solutions of given problem. The existence of Lyapunov function is obtained. The existence of global and trajectory attractors is proved. The relationship between the global attractor, trajectory attractor, and the space of complete trajectories are investigated. We provide that dynamics of all weak solutions of studied problem is finite dimensional within a small parameter. Results of this study allow us to direct the states of investigated system to the desired asymptotic levels, to justify the numerical searching algorithms for weak solutions, and may be used for the development and control of technical equipment based on piezoelectric effect.

Paper reference: (**doi:10.1007/978-3-319-19075-4_16**).

Existence and uniqueness of a solution of an initial problem for a linear differential-difference equation in Banach space at the class of exponential type entire functions

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Let A be a bounded linear operator with a spectral radius $r(A)$ on a complex Banach space X, I be identity operator on X and $h \in \mathbf{C}$, $h \neq 0$.

By an *operator Bruwier series* we understand the following formal operator series

$$\sum_{n=0}^{\infty}(n!)^{-1}(z+nh)^n A^n, \quad z \in \mathbf{C}. \tag{1}$$

Theorem 1. *Let $r(A) < (e|h|)^{-1}$. Then the operator Bruwier series (1) is an entire operator-function of an exponential type $\sigma < |h|^{-1}$.*

Now, we consider an initial problem

$$u'(z) = Au(z+h), \ u(0) = u_0. \tag{2}$$

Theorem 2. *Let $r(A) \leq (2e|h|)^{-1}$. Then for each $u_0 \in X$ the initial problem (2) has the following unique solution*

$$u(z) = \sum_{n=0}^{\infty}(n!)^{-1}(z+nh)^n A^n \Big(I + \sum_{k=1}^{\infty}(k!)^{-1}k^k h^k A^k\Big)^{-1} u_0, \quad z \in \mathbf{C} \quad (3) \quad (3)$$

in the class of entire vector-functions with an exponential type $\sigma < |h|^{-1}$.

Theorem 3. *Let A be a quasinilpotent operator. Then for each $u_0 \in X$ the initial problem (2) has a unique entire solution of zero exponential type. This solution is defined by formula (3).*

On inverse problem of identification of the minor coefficient in ultraparabolic equation

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In the bounded domain $Q_T = \Omega \times D \times (0,T)$, where $\Omega \subset \mathbb{R}^n$, $D \subset \mathbb{R}^l$ and $x \in \Omega,\ y \in D,\ t \in (0,T)$, we consider the inverse problem of identification of a pair of functions $(u(x,y,t), c(t))$ that for almost all $(x,y,t) \in Q_T$ satisfy the equation

$$u_t + \sum_{i=1}^{l} \lambda_i(x,y,t)u_{y_i} - \sum_{i,j=1}^{n} (a_{ij}(x,y,t)u_{x_i})_{x_j} + c(t)u + g(x,y,t,u) = f(x,y,t), \tag{1}$$

and the function u is a weak solution for the initial-boundary value problem for the Eq. (1) and satisfies the overdetermination condition

$$\int_G K(x,y)u(x,y,t)\,dx\,dy = E(t), \ t \in [0,T].$$

The coefficients of the Eq. (1) are bounded and g is Lipschitz continuous function on u.

Using the methods of functional analysis, monotonicity, compactness, and successive approximations, we have determined the sufficient conditions of the unique solvability of the inverse problem in Sobolev spaces.

Integral representations of Brownian functionals

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We generalized the Clark-Ocone stochastic integral representation formula in case, when Brownian functional is not stochastically differentiable in Malliavin sense and established the methods of finding the integrand. At the same time, the functional may depend both on the terminal value and also on the all path of the Brownian motion.

The representation of functionals of Brownian motion by stochastic integrals, also known as martingale representation theorem, states that a functional that is measurable with respect to the filtration generated by a Brownian motion can be written in terms of Ito's stochastic integral with respect to this Brownian motion. The theorem only asserts the existence of the representation and does not help to find it explicitly. It is possible in many cases to determine the form of the representation using Malliavin calculus if a functional is Malliavin differentiable. We consider nonsmooth (in Malliavin sense) functionals and have developed some methods of obtaining constructive martingale representation theorems. The first proof of the martingale representation theorem was implicitly provided by Ito (1951) himself. One of the pioneer work on explicit descriptions of the integrand is certainly the one by Clark (1970). Those of Haussmann (1979), Ocone (1984), Ocone and Karatzas (1991) and Karatzas, Ocone and Li (1991) were also particularly significant.

In many papers using Malliavin calculus or some kind of differential calculus for stochastic processes, the results are quite general but unsatisfactory from the explicitness point of view. Shiryaev and Yor (2003) proposed a method based on Ito's formula to find explicit martingale representation for the running maximum of Brownian motion. In all cases described above investigated functionals, were stochasticaly (in Malliavin sense) smooth. It has turned out that the requirement of smoothness of functional can be weakened by the requirement of smoothness only of its conditional mathematical expectation. We (with prof. O. Glonti) considered Brownian functionals which are not stochastically differentiable. In particular, we generalized the Clark-Ocone formula in case, when functional is not stochastically smooth, but its conditional mathematical expectation is stochastically differentiable and established the method of finding the integrand. Now, we have considered functionals which didn't satisfy even these weakened conditions. To such functionals belong, for example, Lebesgue integral (with respect to time variable) from stochastically non smooth square integrable processes.

1. O. Glonti and O. Purtukhia, *On one integral representation of functionals of Brownian motion*, SIAM J. Theory of Probability and Its Applications, **61**:1 (2017) 133–139.

Approximation of analityc functions by r-repeated de la Vallee Poussin sums

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For upper bounds of the deviations of r-repeated de la Vallee Poussin sums taken over classes of periodic functions that admit analytic extensions to a fixed strip of the complex plane, we obtain asymptotic equalities. In certain cases, these equalities give a solution of the corresponding Kolmogorov-Nikolsky problem. Our main result is contained in the following theorem (see definitions in [1, 2]).

Theorem. *Suppose that $\psi(k) \in D_q$, $q \in (0;1)$, $\psi(k) > 0$, $\beta \in R$, $\omega(t)$ is an arbitrary fixed modulus of continuity. Then the following relations hold as $n - \Sigma_{\overline{p}} \to \infty$, $\Sigma_{\overline{p}} = \sum_{i=1}^{r} p_i$*

$$\mathcal{E}(C^{\psi}_{\beta,\infty}; V^{(r)}_{n,\overline{p}}) = \frac{2\psi(n-\Sigma_{\overline{p}}+r)}{\pi^2 \prod_{i=1}^{r} p_i} e_{n-\Sigma_{\overline{p}}}(\omega) \int\limits_0^{\pi} Z_q^{r+1}(x)dx + O(1)\psi(n-\Sigma_{\overline{p}}+r)\times$$

$$\times\Bigg(\frac{\omega\left([n-\Sigma_{\overline{p}}]^{-1}\right)}{\prod_{i=1}^{r} p_i(n-\Sigma_{\overline{p}})} \left[\frac{1}{(1-q)^{r+3}} + \frac{1}{(1-q)^{2r}}\right] + \frac{\sum_{i=1}^{r} q^{p_i}\omega([n-\Sigma_p^{\alpha(r-1)}]^{-1})}{\prod_{i=1}^{r} p_i(1-q)^{r+1}} +$$

$$+ \frac{\varepsilon_{n-\Sigma_{\overline{p}}+r-1}\,\omega([n-\Sigma_{\overline{p}}+r-1]^{-1})}{(1-q)^2}\Bigg),$$

where

$$Z_q(x) = \frac{1}{\sqrt{1-2q\cos x + q^2}}, \quad e_{n-\Sigma_{\overline{p}}}(\omega) = \theta_n(\omega) \int\limits_0^{\frac{\pi}{2}} \omega(2\tau(n-\Sigma_{\overline{p}})^{-1}) \sin\tau d\tau,$$

$$\varepsilon_{n-\Sigma_{\overline{p}}} = \sup_{k \geq n-\Sigma_{\overline{p}}} \left| \frac{\psi(k+1)}{\psi(k)} - q \right|,$$

$\theta_n(\omega) \in [1/2;1]$, $\theta_n(\omega) = 1$, if $\omega(t)$ is a convex modulus of continuity, $O(1)$ is a quantity uniformly bounded with respect to n, q, β, p_i, $i = 1, 2, \ldots, r$, $\psi(k)$.

1. A. S. Serdyuk, *Approximation of Poisson integrals by de la Vallee Poussin sums*, Ukr. Math. J. **56**:1 (2004), 122–134.
2. V. I. Rukasov, S. O. Chaichenko, *Approximation of the classes of analytical functions by de la Vallee-Poussin sums*, Ukr. Math. J. **55**:6 (2003), 974–986.

Existence of a solution to the inhomogeneous equation with the one-dimensional Schrodinger operator in the space of quickly decreasing functions

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It's known a problem on constructing solutions to the Korteweg-de Vries equation is connected with linear Schrodinger equation being one of the fundamental equation of contemporary physics and mathematics [1, 2].

While studying problem on constructing asymptotic soliton like solutions to the singularly perturbed Korteweg-de Vries equation with variable coefficients [3], a problem on existence of solution in quickly decreasing functions space to differential equation with one-dimension Shrodinger operator arises.

The theorem [4] on necessary and sufficient conditions of existence of solution in quickly decreasing functions space to differential equation with one-dimension Shrodinger operator is proved through the pseudodifferential operators theory.

1. N. J. Zabusky and M. D. Kruskal, *Interaction of 'solitons' in a collisionless plasma and the recurrence of initial states*, Phys. Rev. Lett. **15** (1965), 240–243.
2. C. S. Gardner, J. M. Green, M. D. Kruskal, and R. M. Miura, *Method for solving the Korteweg-de Vries equation*, Phys. Rev. Lett. **19** (1967), 1095–1098.
3. V. Samoylenko and Yu. Samoylenko, *Asymptotic expansions for one-phase soliton-type solutions of the Korteweg-de Vries equation with variable coefficients*, Ukrainian Math. J. **57** (2005), 132–148.
4. V. Samoylenko and Yu. Samoylenko, *Existence of a solution to the inhomogeneous equation with the one-dimensional Schrodinger operator in the space of quickly decreasing functions*, J. Math. Sci. **187** (2012), 70–76.

On the effect of the fixed points of some types of a certain function

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Some types of a strictly increasing continuous function $\beta : I \subseteq \mathbb{R} \to I$, where $\beta(t) \in I$ for every $t \in I$, are given. Each type contains a certain number of fixed points. A study is given on the effect of each fixed point $s_0 \in I$ on the direction of the movement of $\beta^k(t)$ as $k \in \mathbb{N}$ tends to infinity and t belongs to the neighborhood of s_0, where $\beta^k(t) = \underbrace{\beta \circ \beta \cdots \circ \beta(t)}_{k-times}$.

Problem with integral condition for nonhomogeneous equation with Gelfond-Leontiev generalized differentiation

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Denote by S_α^α, $\alpha \in \left(\frac{1}{2}, 1\right)$, the Gelfand-Shilov space [1] of all functions $\varphi \in C^\infty(\mathbb{R})$ that admit analytic continuation into the complex plane to an entire function $\varphi(x+iy)$, satisfying

$$\exists\, a, b, c > 0 \quad \forall z = x + iy \in \mathbb{C} \quad |\varphi(z)| \le c \exp\left(-a|x|^{1/\alpha} + b|y|^{1/\alpha}\right).$$

Let $f(z) = \sum\limits_{k=0} f_k z^k$ be an entire function with all positive coefficients $f_k > 0$, $k \ge 0$, and for fixed $m \in \mathbb{N}$

$$\exists\, c > 0 \quad \exists\, L > 1 \quad \forall\, k \ge m \quad \left| \frac{f_{k-m}}{f_k} \right| \le c L^k.$$

For a function $\varphi \in S_\alpha^\alpha$, $\varphi(x) = \sum\limits_{k=0}^{\infty} \varphi_k x^k$, the operation

$$D_{f(x)}^m \varphi(x) = \sum_{k=m}^{\infty} \varphi_k \frac{f_{k-m}}{f_k} x^{k-m},$$

is said to be a Gelfond-Leontiev generalized differentiation of φ with respect to the function f [2]. The generalized differentiation operator $D_{f(x)}^m$ is well-defined on S_α^α, $\alpha \in \left(\frac{1}{2}, 1\right)$, for arbitrary fixed $m \in \mathbb{N}$ and continuously maps this space into itself [3].

The report is devoted to the presentation of the results obtained in the study (in the spaces S_α^α, $\alpha \in \left(\frac{1}{2}, 1\right)$) of such integral problem

$$\partial_t u(t,x) = D_{f(x)}^m u(t,x) + g(t,x), \qquad \int_0^T e^{-at} u(t,x)\,dt = \varphi(x), \quad a, T > 0.$$

1. I. M. Gelfand and G. E. Shilov, *Generalized Functions, II, III,* Academic Press, New York-London, 1967.
2. A. O. Gelfond and A. F. Leontiev, *On a generalization of the Fourier series*, Mat. Sb. **29** (1951), 477–500.
3. V. V. Gorodetsky and O. V. Martynyuk, *Gelfond-Leontiev generalized differentiation operators in spaces of type S*, Sib. Math. J. **54**:3 (2013), 446–458.

Approximation in weighted Lebesgue space with variable exponent

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In this talk the direct and inverse theorems of approximation theory in the weighted variable exponent Lebesgue spaces in the term of the integer order modulus of smoothness are discussed. Let $\mathbb{T} := [0, 2\pi]$ and let $p(\cdot) : \mathbb{T} \to [0, \infty)$ be a Lebesgue measurable 2π periodic function. The variable exponent Lebesgue space $L^{p(\cdot)}(\mathbb{T})$ is defined as the set of all Lebesgue measurable 2π periodic functions f such that $\rho_{p(\cdot)}(f) := \int_0^{2\pi} |f(x)|^{p(x)}dx < \infty$. In this work we suppose that the considered exponent functions $p(\cdot)$ satisfy the conditions :

$$1 < p_- := ess \inf_{x\in\mathbb{T}} p(x) \le ess \sup_{x\in\mathbb{T}} p(x) := p^+ < \infty,$$

$$|p(x) - p(y)| \ln(1/|x-y|) \le c,\ \ x, y \in \mathbb{T},\ \ 0 < |x-y| \le 1/2.$$

The class of these exponents is denoted by $\mathcal{P}_0(\mathbb{T})$. Equipped with the norm $\|f\|_{p(\cdot)} := \inf\{\lambda > 0 : \rho_{p(\cdot)}(f/\lambda) \le 1\}$, $L^{p(\cdot)}(\mathbb{T})$ becomes a Banach space.

Let ω be a weight function on $\mathbb{T}$, i.e. an almost everywhere positive and Lebesgue integrable function on $\mathbb{T}$. For a given weight ω we define the weighted variable exponent Lebesgue space $L^{p(\cdot)}_\omega(\mathbb{T})$ as the set of all measurable functions f on $\mathbb{T}$ such that $f\omega \in L^{p(\cdot)}(\mathbb{T})$. The norm of $f \in L^{p(\cdot)}_\omega(\mathbb{T})$ can be defined as $\|f\|_{p(\cdot),\omega} := \|f\omega\|_{p(\cdot)}$. If $\sup_{I\subset\mathbb{T}} |I|^{-1}\|\omega\chi_I\|_{p(\cdot)}\|\omega^{-1}\chi_I\|_{q(\cdot)} < \infty$, where $1/p(\cdot)+1/q(\cdot) = 1$ and $|I|$ is the Lebesgue measure of the interval $I \subset \mathbb{T}$, with the characteristic function χ_I, then we say that $\omega \in A_{p(\cdot)}(\mathbb{T})$. For $f \in L^{p(\cdot)}_\omega(\mathbb{T})$ we define the best approximation number $E_n(f)_{p(\cdot),\omega} := \inf\left\{\|f - T_n\|_{p(\cdot),\omega} : T_n \in \Pi_n\right\}$ in the class Π_n of the trigonometric polynomials of degree not exceeding n. The variable exponent Sobolev space $W^{p(\cdot)}_{\omega,k}(\mathbb{T})$, where $k = 1, 2, \ldots$, is defined as $\{f : f^{(k-1)}$ is absolutely continuous and $f^{(k)} \in L^{p(\cdot)}_\omega(\mathbb{T})\}$.

Definition 1. Let $f \in L^{p(\cdot)}_\omega(\mathbb{T})$, $p(\cdot) \in \mathcal{P}_0(\mathbb{T})$, $\omega(\cdot) \in A_{p(\cdot)}(\mathbb{T})$ and let

$$\Delta_t^k f(x) := \sum_{s=0}^{k} (-1)^{k+s} \binom{k}{s} f(x + st),\ \ k = 1, 2, \ldots .$$

We define the r th modulus of smoothness as

$$\Omega_k(f,\delta)_{p(\cdot),\omega} := \sup_{|h|\le\delta} \left\| \frac{1}{h} \int_0^h \Delta_t^k f(x)dt \right\|_{p(\cdot),\omega}, \quad \delta > 0.$$

By $c(\cdot,\cdot)$, $c(\cdot,\cdot,\cdot)$ we denote the constants depending in general of the parameters given in the brackets but independent of n. Main results discussed in this talk are following:

Theorem 1. *If $f \in W^{p(\cdot)}_{\omega,r}(\mathbb{T})$, $p(\cdot) \in \mathcal{P}_0(\mathbb{T})$, $\omega \in A_{p(\cdot)}(\mathbb{T})$ and $r = 1,2,\ldots$, then*

$$E_n(f)_{p(\cdot),\omega} \le \frac{c(p,r)}{n^r} E_n(f^{(r)})_{p(\cdot),\omega}, \quad n = 1,2,\ldots$$

Theorem 2. *If $f \in W^{p(\cdot)}_{\omega,r}(\mathbb{T}), p(\cdot) \in \mathcal{P}_0(\mathbb{T})$, $\omega(\cdot) \in A_{p(\cdot)}(\mathbb{T})$ and $k = 1,2,...$, then for any $r = 0,1,2,\ldots$*

$$E_n(f)_{p(\cdot),\omega} \le \frac{c(p,r,k)}{n^r} \Omega_k(f^{(r)},1/n)_{p(\cdot),\omega}, \quad n = 1,2,\ldots$$

The following theorem is the inverse of Theorem 2 in the case of $r = 0$.

Theorem 3. *If $f \in L^{p(\cdot)}_{\omega}(\mathbb{T})$, $p(\cdot) \in \mathcal{P}_0(\mathbb{T})$, $\omega(\cdot) \in A_{p(\cdot)}(\mathbb{T})$ and $k = 1,2,3,...$, then*

$$\Omega_k(f,1/n)_{p(\cdot),\omega} \le \frac{c(p,k)}{n^k} \sum_{m=0}^{n} (m+1)^{k-1} E_m(f)_{p(\cdot),\omega}, \quad n = 1,2,...$$

In the case of $p_- \ge 1$ these results were proved for $L^{p(\cdot)}(\mathbb{T})$ in [1].

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1. D. M. Israfilov and A. Testici, Approximation Problems in the Lebesgue Spaces with Variable Exponent, (submitted for publication).

Integrability of minimizers of variational high order integrals with strengthened coercivity

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In this talk, we present a result on the improved integrability of minimizers of the variational Dirichlet problem for integral functionals whose integrands depend on high order derivatives of an unknown function and satisfy a strengthened coercivity condition.

Let $m, n \in \mathbb{N}$, $p, \overline{p}, q \in \mathbb{R}$ be numbers such that $m \geqslant 3, n{>}2(m-1)$, $\frac{2n(m-2)}{n(m-1)-2}{<}p{<}\frac{n}{m}$, $\overline{p} = 2p/[p(m-1) - 2(m-2)]$, $\max(\overline{p}, mp){<}q{<}n$. Next, let Ω be a bounded open set of $\mathbb{R}^n$. We denote by $W^{1,q}_{m,p}(\Omega)$ the Banach space $W^{1,q}(\Omega) \cap W^{m,p}(\Omega)$ with the norm $\|u\| = \|u\|_{W^{1,q}(\Omega)} + \big(\sum\limits_{|\alpha|=m} \int_\Omega |D^\alpha u|^p dx\big)^{1/p}$. $\overline{W}^{1,q}_{m,p}(\Omega)$ is the closure of $C_0^\infty(\Omega)$ in $W^{1,q}_{m,p}(\Omega)$, and $\mathbb{R}^{n,m}$ is the space of all sets $\xi = \{\xi_\alpha : 1 \leqslant |\alpha| \leqslant m\}$ of real numbers. For every $u \in W^{1,q}_{m,p}(\Omega)$, we set $\nabla_m u = \{D^\alpha u : 1 \leqslant |\alpha| \leqslant m\}$.

Let $F : \Omega \times \mathbb{R}^{n,m} \to \mathbb{R}$ be a Caratheodory function such that for a.e. $x \in \Omega$ the function $F(x, \cdot)$ is convex in $\mathbb{R}^{n,m}$ and for a.e. $x \in \Omega$ and every $\xi \in \mathbb{R}^{n,m}$,

$$F(x,\xi) \geqslant c_1\Big(\sum_{|\alpha|=1} |\xi_\alpha|^q + \sum_{|\alpha|=m} |\xi_\alpha|^p\Big) - c_2 \sum_{2\leqslant|\alpha|\leqslant m-1} |\xi_\alpha|^{\tilde{q}_\alpha} - f_1(x), \quad (1)$$

$$F(x,\xi) \leqslant c_3\Big(\sum_{|\alpha|=1} |\xi_\alpha|^q + \sum_{|\alpha|=m} |\xi_\alpha|^p + \sum_{2\leqslant|\alpha|\leqslant m-1} |\xi_\alpha|^{\tilde{q}_\alpha}\Big) + f_2(x),$$

where $c_{1,2,3} > 0$, $f_{1,2} \in L^1(\Omega)$, $f_{1,2} \geqslant 0$, $\tilde{q}_\alpha \in (1, q_\alpha)$ and $q_\alpha^{-1} = (|\alpha| - 1)/p(m-1) + (m - |\alpha|)/q(m-1)$.

We set $q^* = nq/(n-q)$, and let $f \in L^{q^*/(q^*-1)}(\Omega)$. Let $I : \overline{W}^{1,q}_{m,p}(\Omega) \to \mathbb{R}$ be the functional such that

$$\forall u \in \overline{W}^{1,q}_{m,p}(\Omega),\ I(u) = \int_\Omega \{F(x, \nabla_m u) + fu\} dx.$$

Theorem 1. *Let $r > q^*/(q^*-1)$, f, $f_{1,2} \in L^r(\Omega)$, and let u be a function minimizing I on $\overline{W}^{1,q}_{m,p}(\Omega)$. Then the following assertions hold:*

1) *if $r{<}n/q$, then $u \in L^\lambda(\Omega)$ for every $\lambda \in \big(q^*, nr(q-1)/(n-qr)\big)$;*
2) *if $r = n/q$, then $u \in L^\lambda(\Omega)$ for every $\lambda \geqslant 1$;*
3) *if $r{>}n/q$, then $u \in L^\infty(\Omega)$.*

Remark 1. The strengthened coercivity condition like (1) goes back to Skrypnik [1], used because of the regularity problem of generalized solutions to high order elliptic partial differential equations. For solutions of high order equations with strengthened coercivity, the results analogous to Theorem 1 were proved in [2], [3]. The proofs are based on the development of Stampacchia method proposed in [4] for second-order equations.

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1. I. V. Skrypnik, *On quasilinear elliptic higher order equations with continuous generalized solutions*, Differential Equations **14**:6 (1978), 786–795.
2. A. A. Kovalevskii and M. V. Voitovich, *On increasing the integrability of generalized solutions of the Dirichlet problem for fourth-order nonlinear equations with strengthened ellipticity*, Ukr. Math. J. **58**:11 (2006), 1717–1733.
3. M. V. Voitovich, *Integrability properties of generalized solutions of the Dirichlet problem for higher-order nonlinear equations with strengthened ellipticity*, Trudy Inst. Prikl. Mat. Mekh. Nat. Akcad. Nauk Ukr. **15** (2007), 3–14.
4. G. Stampacchia, *Régularisation des solutions de problèmes aux limites elliptiques à données discontinues*, Proc. Internat. Sympos. Linear Spaces (Jerusalem, 1960), Pergamon, Oxford, (1961), 399–408.

Weighted approximation by Picard operators depending on nonisotropic beta distance

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In this work, for a weight depending on nonisotropic distance, we give definitions of n-dimensional nonisotropic weighted-L_p space and nonisotropic weighted beta Lebesgue point at which we obtain a pointwise convergence result for the family of $P(f)$ for f belonging to this weighted space. We also give the measure of the rate of this pointwise convergence. Convergence in the norm of this space is also discussed.

The consistent criteria of checking hypotheses for stationary statistical structures

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Let given the real Gaussian stationary process with zero means

$$M(\xi_i(t)\xi_i(s)) = R_i(t-s)$$

and correlation functions, $t \in T$, $i \in I$, where T is closed bounded subset of the axis of real numbers R and I is the set of hypotheses. Let $\{\mu_i : i \in I\}$ be corresponding probability measures given on (E, S) and $f_i(\lambda)$, $\lambda \in R$, $i \in I$ be their spectral densities. Assume that for every $i \in I$ $\int_R \frac{\tilde{b}_i(\lambda))^2}{f_i(\lambda)} d\lambda = \pm\infty$, where $\tilde{b}_i(\lambda)$ is the generalized Fourier transformation of the functions $b_{ij}(t) = R_i(t) - R_i(j)$, $i, j \in I$. Then the corresponding probability measures μ_i and μ_j are pairwise orthogonal and $\{E, S, \mu_i : i \in I\}$ are Gaussian orthogonal statistical structures.

Theorem. *Let $M_H = \oplus_{i\in I} H_2(\mu_i)$ be the Hilbert of measures (see[1,2]), where $H_2(\mu_i)$ is the family of measures $\nu(A) = \int_A f(x)\mu_i(dx)$, $\forall A \in S$, $\int_E |f(x)|^2 \mu_i(dx) < +\infty$; E be the complete separable metric space, S be the Borel σ-algebra in E and $cardI \leq 2^{\aleph_0}$. Then in the theory ZFC+MA the family of Gaussian orthogonal statistical structures $\{E, S, \mu_i : i \in I\}$ admits a consistent criteria of hypotheses if and only if correspondence $f \leftrightarrow \phi_f$, given by the equality $\int_E f(x)\nu(x) = (\phi_f, \nu)$, $\nu \in M_H$, is one-to-one.*

1. Z. Zerakidze. *Hilbert space of measures*, Ukr. Math. J. **38**:2 (1986), 147–153.
2. Z. Zerakidze and M. Mumladze, *Statistical structures and consistent criteria for checking hypotheses*, Lambert Academic Publishing, Saarbruken, Deutschland, (2015).

Section: Complex Analysis

Analytic in a ball functions of bounded **L**-index in joint variables

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We consider analytic in the unit ball functions of bounded **L**-index in joint variables. We use the definitions and notations from [1].

The following theorem is basic in the theory of functions of bounded index. It is necessary to prove more efficient criteria of index boundedness which describe behavior of the maximum modulus on a disc or behavior of the logarithmic derivative (see [2, 3, 4, 5]).

Theorem. *Let* $\mathbf{L} \in Q(\mathbb{B}^n)$*. An analytic function* F *in* $\mathbb{B}^n$ *has bounded* $\mathbf{L}$*-index in joint variables if and only if for each* $R \in \mathbb{R}^n_+$, $|R| \leq \beta$, *there exist* $n_0 \in \mathbb{Z}_+$, $p_0 > 0$ *such that for every* $z^0 \in \mathbb{B}^n$ *there exists* $K^0 \in \mathbb{Z}^n_+$, *such that* $\|K^0\| \leq n_0$, *and*

$$\max\left\{\frac{|F^{(K)}(z)|}{K!\,\mathbf{L}^K(z)} \colon \|K\| \leq n_0,\ z \in \mathbb{D}^n[z^0, R/\mathbf{L}(z^0)]\right\} \leq p_0 \frac{|F^{(K^0)}(z^0)|}{K^0!\,\mathbf{L}^{K^0}(z^0)}.$$

1. A. Bandura and O. Skaskiv, *Analytic in a ball functions of bounded* **L***-index in joint variables*, **https://arxiv.org/abs/1705.09568**.
2. A. I. Bandura and O. B. Skaskiv, *Entire functions of bounded L-index in direction*, Mat. Stud. **27**:1 (2007), 30–52 (in Ukrainian).
3. A. Bandura and O. Skaskiv, *Entire functions of several variables of bounded index*, Publisher I. E. Chyzhykov, Lviv, 2016 (**http://chyslo.com.ua**).
4. M. Sheremeta, *Analytic functions of bounded index*, VNTL Publishers, Lviv, 1999.
5. V. O. Kushnir and M. M. Sheremeta, *Analytic functions of bounded l-index*, Mat. Stud. **12**:1 (1999), 59–66.

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Entire functions of zero order with v-density of zeros on logarithmic spirals

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Let L be the set of growth functions v such that $rv'(r) = o(v(r))$ as $r \to \infty$; $H_0(v)$ be the class of entire functions f of zero order that satisfy the condition $n(r) = O(v(r))$, $r \to \infty$, where $n(r) = n(r, 0, f)$ is the counting function of zeros (a_n) of the function f. For a real number c we denote $L^c_\varphi(r) = \{z : z = te^{i(\varphi + c\ln t)}, 1 \le t \le r\}$, $L^c_\varphi(+\infty) = L^c_\varphi$ is logarithmic spiral, $D^c_r(\alpha, \beta) = \bigcup\limits_{\alpha \le \varphi \le \beta} L^c_\varphi(r)$. Let $n^c(r, \alpha, \beta)$ be the number of zeros of the function f in the curvilinear sector $D^c_r(\alpha, \beta)$.

We say that zeros of the function $f \in H_0(v)$ have v-density $\Delta^c(\alpha, \beta)$ on logarithmic spirals L^c_φ if the limit $\lim_{r\to\infty} n^c(r, \alpha, \beta)/v(r) = \Delta^c(\alpha, \beta)$ exists for all $0 \le \alpha < \beta < 2\pi$, with the exception, perhaps, some countable set of φ.

The connection between existence of v-density $\Delta^c(\alpha, \beta)$ of zeros of the function $f \in H_0(v), v \in L$ on L^c_φ and asymptotic behavior of $\ln f(re^{i(\varphi + c\ln r)})$ as $r \to +\infty$ outside C_0-set is studied.

Sandwich results for p-valent meromorphic functions associated with Hurwitz-Lerch zeta function

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Using the principle of subordination, in the present paper we obtain the sharp subordination and superordination-preserving properties of some convex combinations associated with a linear operator in the open unit disk. The sandwich-type theorem on the space of meromophic functions for these operators is also given, together with a few interesting special cases obtained for appropriate choices of the parameters and the corresponding functions.

Non-regular solutions of complex differential equations in the unit disc

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We consider growth properties of solutions of the differential equation

$$f^{(k)}(z) + A(z)f(z) = 0$$

where $k \in \mathbb{N}$, A is analytic in a domain of the complex plane. It is well known that if A is a polynomial of degree n then the order of any nontrivial solution is equal to $n/k + 1$, and the lower order is the same. The situation is much more complicated when A is analytic in the unit disc. There are a lot of results that connect the growth of the coefficient A and the order of a solution, but nothing is known about its lower order.

We show that 'non-regularity' of a coefficient always implies that the lower order is smaller than the order, and give some estimates for the lower order of solutions.

Joint work with Jouni Rättyä and Janne Gröhn.

Solvability criterion for the convolution equation in half-strip

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Let $E^p[D_\sigma]$ and $E^p_*[D_\sigma], 1 \le p < +\infty, \sigma > 0$, be the spaces of analytic functions in the half-strip $D_\sigma = \{z : |\mathrm{Im} z| < \sigma, \mathrm{Re} z < 0\}$ and $D^*_\sigma = \mathbb{C}\backslash\overline{D}_\sigma$, respectively, satisfying

$$\|f\| := \sup\Big\{\int\limits_\gamma |f(z)|^p |dz|\Big\}^{1/p} < +\infty,$$

where supremum is taken over all segments γ, that lay in D_σ and D^*_σ, respectively. For $\sigma > 0$ we consider the convolution equation

$$\int\limits_{\partial D\sigma} f(w+\tau)g(w)dw = 0,\ \tau \leqslant 0,\ g \in E^2_*[D_\sigma]. \tag{1}$$

An analogue of the above equation for $\sigma = 0$ is

$$\int_{-\infty}^{0} f(w+\tau)g(w)dw = 0, \tau \leqslant 0, \quad g \in L^2(-\infty; 0).$$

Let $H^p_\sigma(\mathbb{C}_+)$ be the (weighted Hardy) space of holomorphic in $\mathbb{C}_+$ functions f, such that

$$\sup_{\varphi\in(-\frac{\pi}{2};\frac{\pi}{2})}\Big\{\int_0^{+\infty}|f(re^{i\varphi})|^p e^{-p\sigma r|\sin\varphi|}dr\Big\}^{1/p}<+\infty.$$

Theorem. *A function $f\in E^2[D_\sigma]$, $\sigma\ge 0$, is the solution of equation (1) if and only if the function $\Phi(iy):=F(iy)G(iy)$, where*

$$F(z)=\frac{1}{\sqrt{2\pi}}\int_{-i\sigma-\infty}^{-i\sigma}f(w)e^{-zw}dw,\qquad G(z):=\frac{1}{i\sqrt{2\pi}}\int_{\partial D_\sigma}g(w)e^{zw}dw$$

is the angular boundary function of some function P such that $P(z)e^{-i\sigma z}\in H^1_\sigma(\mathbb{C}_+)$.

Analogues of Whittaker's theorem for Laplace-Stieltjes integrals

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For an entire function $g(z)=\sum\limits_{n=0}^{\infty}a_n z^{\lambda_n}$ let ϱ and λ be the order and the lower order of g correspondingly. J.M. Whittaker (1933) has proved that $\lambda\le\varrho\beta$, where $\beta=\varliminf\limits_{n\to+\infty}(\ln\lambda_n)/\ln\lambda_{n+1}$. If g is an analytic in $\{z:\ |z|<1\}$ function of the order ϱ_0 and the lower order λ_0 then L.R. Sons (1968, 1977) proved that an analogue of Whittaker's theorem is valid, i.e., $\lambda_0\le\rho_0\beta$ (the first version of her paper contains a mistake). P.V. Filevych and M.M. Sheremeta (2006) obtained analogues of Whittaker's theorem for Dirichlet series. Here we investigate similar problems for the integrals of Laplace-Stieltjes.

Let V be the class of nonnegative functions F that nondecreasing unbounded and continuous on the right on $[0,+\infty)$. For a nonnegative function f on $[0,+\infty)$ the integral $I(\sigma)=\int\limits_0^{\infty}f(x)e^{x\sigma}dF(x)$ is called of Laplace-Stieltjes. Let $\mu(\sigma,I)=\sup\{f(x)e^{x\sigma}:\ x\ge 0\}$ be the maximum of the integrand and σ_μ be its abscissa of the existence. We will say that a nonnegative function f has regular variation in regard to F if there exist $a\ge 0$, $b\ge 0$ and $h>0$ such that $\int\limits_{x-a}^{x+b}f(t)dF(t)\ge hf(x)$ for all $x\ge a$.

The most used characteristics of growth for integrals $I(\sigma)$ with σ_μ=$+\infty$ are R-order $\varrho_R[I]$, lower R-order $\lambda_R[I]$ and (if $\varrho_R[I]\in(0,+\infty)$) R-type $T_R[I]$, lower R-type $t_R[I]$, which are defined by formulas

$$\varrho_R[I]=\varlimsup_{\sigma\to+\infty}\frac{\ln\ln I(\sigma)}{\sigma},\quad \lambda_R[I]=\varliminf_{\sigma\to+\infty}\frac{\ln\ln I(\sigma)}{\sigma},$$

$$T_R[I] = \varlimsup_{\sigma\to+\infty} \frac{\ln I(\sigma)}{\exp\{\sigma\varrho_R[I]\}}, \quad t_R[I] = \varliminf_{\sigma\to+\infty} \frac{\ln I(\sigma)}{\exp\{\sigma\varrho_R[I]\}}.$$

The following theorem is true.

Theorem. *Let $F \in V$, $\sigma_\mu = +\infty$ and $X = (x_k)$ be a sequence $X = (x_k)$ of positive numbers increasing to $+\infty$. Suppose that f is nonincreasing and has regular variation in regard to F.*

If $\ln F(x) = O(x)$ *as* $x \to +\infty$ *and* $\ln f(x_k) = (1 + o(1)) \ln f(x_{k+1})$ *as* $k \to \infty$ *then*

$$\lambda_R[I] \le \beta\varrho_R[I], \quad \beta = \varliminf_{k\to\infty} \frac{\ln x_k}{\ln x_{k+1}}.$$

If $\ln F(x) = o(x)$ *as* $x \to +\infty$ *and* $\ln f(x_k) - \ln f(x_{k+1}) = O(1)$ *as* $k \to \infty$ *then*

$$t_R[I] \le T_R[I] \frac{\gamma}{1-\gamma} \exp\left\{1 + \frac{\gamma\ln\gamma}{1-\gamma}\right\} \ln\frac{1}{\gamma}, \quad \gamma = \varliminf_{k\to\infty} \frac{x_k}{x_{k+1}}.$$

An analogous theorem is valid for the case $\sigma_\mu = 0$.

Classes of harmonic functions defined by subordination

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Harmonic functions are famous for their use in the study of minimal surfaces and also play important roles in a variety of problems in applied mathematics. Recent interest in harmonic complex functions has been triggered by geometric function theorists Clunie and Sheil-Small [1].

A complex-valued continuous function $f : D \to \mathbb{C}$ is said to be harmonic in $D \subset \mathbb{C}$ if both functions $u := \operatorname{Re} f$ and $v := \operatorname{Im} f$ are real-valued harmonic functions in D. In any simply connected domain D, we can write $f = h + \overline{g}$, where h and g are analytic in D.

Let $\mathcal{H}$ denote the family of complex-valued functions which are harmonic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. We consider the usual topology on $\mathcal{H}$ defined by a metric in which a sequence in $\mathcal{H}$ converges to f if it converges to f uniformly on each compact subset of $\mathbb{U}$. It follows from the theorems of Weierstrass and Montel that this topological space is complete.

The object of the present talk is to define and study some subclasses of $\mathcal{H}$ which are convex and compact subsets of $\mathcal{H}$. By using extreme points theory we obtain solutions of extremal problems in the defined classes of functions.

1. J. Clunie and T. Sheil-Small, *Harmonic univalent functions*, Ann. Acad. Sci. Fenn. Ser. A I Math. **9** (1984), 3–25.

One property of exponential sums in the Stepanov's metric

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Let $\{(a_n, \lambda_n)\}_{n=1}^{\infty}$, $a_n \in \mathbb{C}$, $\lambda_n \in \mathbb{R}^d$, $S_N(x) = \sum_{k=1}^{N} a_k e^{i\langle\lambda_k, x\rangle}$ and $\Lambda = \bigsqcup_{j=1}^{\infty} \Lambda_j$ be a partition of the set $\Lambda = \{\lambda_n\}_{n=1}^{\infty}$ with the property $\operatorname{diam} \Lambda_j \leq 1$, $j = 1, 2, \ldots$. Let $B(x_j, 2)$ be a set of balls with uniformly bounded multiplicities of intersections such that $\{\Lambda_j\} \subset B(x_j, 2)$ for all $j \in \mathbb{N}$.

Under above conditions if $\sum\limits_{j=1}^{\infty} \big(\sum\limits_{\lambda_n \in \Lambda_j} |a_n| \big)^2 = K^2$ for some $a_n \in \mathbb{C}$, then

$$\sup_{y \in \mathbb{R}^d} \left[\int\limits_{B(y,1)} |S_N(x)|^2 dx \right]^{\frac{1}{2}} < C \cdot K,$$

where C does not depend on N.

Mappings with integrally controlled p-moduli

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It is the well known fact that the class of conformal mappings of the multidimensional Euclidean spaces exhausts only Möbius transformations. This strong rigidity has forced to introduce various classes of mappings which preserve in some sense the main features of analyticity in the plane. Then the class of quasiconformal mappings (homeomorphisms of the Sobolev space $W^{1,n}$ with uniformly bounded distortion coefficient $K(x)$) has been defined first in the plane and later in $\mathbb{R}^n$. It is well known that such mappings have admit rich analytic and geometric properties like differentiability almost everywhere, preservation of sets of zero measure (Lusin's (N) and (N^{-1})-properties), etc. Nonhomeomorphic quasiconformal mappings are called quasiregular or mappings with bounded distortion (following Reshetnyak). The next essential extension of quasiconformality and quasiregularity relates to the class of mappings of finite distortion. Here the uniform boundedness of the distortion function is relaxed by finiteness almost everywhere. All these classes have been defined by involving a purely analytical approach.

One of the most fruitful tools for studying various properties of mappings relies to the module method which goes back to the classical papers by Ahlfors-Beurling and Fuglede. A crucial point here that in a contrast to other methods it is easily applied for higher dimensions. In the talk, we represent the classes of mappings in

$\mathbb{R}^n$ related to p-moduli with prescribed upper integral bounds. We survey the known results in this field and present new ones regarding to the geometric, differentiable and topological properties of such mappings.

The results are based on joint papers with Ruslan Salimov, Institute of Mathematics of the National Academy of Sceinces of Ukraine.

On a local Darlington synthesis problem

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The Darlington synthesis problem (in the scalar case) is the problem of embedding a given analytic on the unit disk $\mathbf{D}$ and contractive function s to an inner 2×2 matrix function $S = \|s_{ij}\|_{i,j=1}^2$ as an entry: $s_{22} = s$. A fundamental result of Arov–Douglas–Helton relates this algebraic property to a purely analytic one known as a pseudocontinuation of bounded type across the unit circle $\mathbf{T}$. We suggest a local version of the Darlington synthesis problem, that is, the above matrix function is supposed to be contractive inside $\mathbf{D}$ and unitary a.e. on an arc $\gamma \subset \mathbf{T}$, and prove a local analog of the Arov–Douglas–Helton theorem.

On the intersection of weighted Hardy spaces

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Let $H^p(\mathbb{C}_+)$, $1 \leqslant p < +\infty$, be the Hardy space of holomorphic in $\mathbb{C}_+ = \{z : \mathrm{Re} z > 0\}$ functions f such that

$$||f||^p = \sup_{x>0} \Big\{ \int_{-\infty}^{+\infty} |f(x+iy)|^p dy \Big\} < +\infty.$$

Let $H^p_\sigma(\mathbb{C}_+)$ $1 \leq p < +\infty$, $0 \leq \sigma < +\infty$, be the space of all functions f analytic in the half plane $\mathbb{C}_+$ and such that

$$\|f\| := \sup_{\varphi \in (-\frac{\pi}{2};\frac{\pi}{2})} \Big\{ \int_0^{+\infty} |f(re^{i\varphi})|^p e^{-p\sigma r|\sin\varphi|} dr \Big\}^{1/p} < +\infty.$$

The interest to the space $H^p_\sigma(\mathbb{C}_+)$ is generated by studies of completeness, by the theory of integral operators, and the shift operator.

A number of papers has been devoted to the intersection of Hardy and related spaces (see [1, 2]). The aim of our research is to describe some properties of the following space

$$H^p_\cap(\mathbb{C}_+) = \bigcap_{\sigma>0} H^p_\sigma(\mathbb{C}_+).$$

Obviously, $H^p_\cap(\mathbb{C}_+) \supset H^p(\mathbb{C}_+)$ and $H^p_\cap(\mathbb{C}_+) \subset H^p_\varepsilon(\mathbb{C}_+)$ for all ε.

Theorem. $H^p_\cap(\mathbb{C}_+) \neq H^p(\mathbb{C}_+)$.

1. S. I. Fedorov, *On a recent result on the intersection of weighted Hardy spaces*, New Zealand J. Math. **30**:2 (2001), 119–130.
2. P. W. Jones, *Recent advances in the theory of Hardy spaces*, Proc. Intern. Congress of Math., August 16-24, 1983, Warszawa.

Starlike, convex and close-to-convex Dirichlet series

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Let $\{\lambda_n\}_{n=0}^\infty$ be a strictly increasing sequence of real numbers such that $\lambda_n > 0$ for all $n \in \mathbb{N}$. We will denote by SD_0 the class of functions G that are analytic in the half-plane $\Pi_0 = \{s : \mathrm{Re} s < 0\}$ and can be represented by the Dirichlet series

$$G(s) = \exp(s\lambda_1) + \sum_{k=2}^{\infty} g_k \exp(s\lambda_k).$$

Theorem 1. *No function in the class SD_0 is univalent in Π_0. But there exist functions $G \in SD_0$ that are conformal mappings of Π_0, for instance, it is true if $\sum\limits_{k=2}^{\infty} \lambda_k |g_k| \le \lambda_1$.*

Let G be a conformal mapping in Π_0. We say that G is *starlike* provided $\mathrm{Re}\,\frac{G'(s)}{G(s)} > 0$ for all $s \in \Pi_0$. A function G is *convex* if $\mathrm{Re}\,\frac{G''(s)}{G'(s)} > 0$ for all $s \in \Pi_0$.

Theorem 2. *If $\sum\limits_{k=2}^{\infty} \lambda_k |g_k| \le \lambda_1$, then G is starlike. If $\sum\limits_{k=2}^{\infty} \lambda_k^2 |g_k| \le \lambda_1^2$, then G is convex in Π_0.*

The conformal mapping G is called close-to-convex in Π_0, if there exists a convex function $\Psi \in SD_0$ such that $\mathrm{Re}\,\frac{G'(s)}{\Psi'(s)} > 0$ for all $s \in \Pi_0$.

The following theorem is an analogue of Alexander's theorem dealing with analytic functions in the unit disc with positive Taylor coefficients.

Theorem 3. *Let $\lambda_k = \lambda_{k-1} + \lambda_1$ and $g_k > 0$ for all $k \ge 2$. If*

$$\lambda_1 \ge \lambda_2 g_2 \ge ... \ge \lambda_k g_k \ge \lambda_{k+1} g_{k+1} \ge \dots,$$

then the function G is close-to-convex in Π_0.

Let us consider the differential equation

$$\frac{d^2w}{ds^2} + (\gamma_0 e^{2hs} + \gamma_1 e^{hs} + \gamma_2)w = 0$$

with real parameters γ_0, γ_1, γ_2 and h. We established conditions on the parameters under which the equation admits a solution, which is starlike, convex or close-to-convex Dirichlet series. For example, if h is positive, γ_j are negative for $j = 0, 1, 2$, $|\gamma_1| \le \frac{2\sqrt{|\gamma_2|}+h}{\sqrt{|\gamma_2|}+h} h\sqrt{|\gamma_2|}$ and

$$|\gamma_0| \le \left(\frac{4h(\sqrt{|\gamma_2|}+h)^2}{\sqrt{|\gamma_2|}+2h} - |\gamma_1| \right) \frac{|\gamma_1|}{h(2\sqrt{|\gamma_2|}+h)},$$

then the differential equation has an entire solution which is a Dirichlet series, close-to-convex in Π_0.

On correspondence of ratios for hypergeometric functions of several variables to their expansions into branched continued fractions

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Functional continued fractions are important in approximation of special classes of analytic functions. Attention to continued fractions is due in particular to the fact that in some cases the convergence domain of such fraction is wider than the domain of the corresponding power series [1, 2].

Branched continued fractions (BCF) are used to construct rational approximations of functions of several variables, in particular, to approximate the ratios of hypergeometric functions of several variables. In this case, we need to be solve the following problems:

1) to construct the expansions of ratios for hypergeometric function into the BCF;
2) to investigate the convergence of these expansions;
3) to prove that the BCF converges to the same function for which expansion was constructed.

To solve these problems, we use the correspondence to the given multiple power series, as well as recurrence relations. So, for the constructed expansions of the ratios of the hypergeometric Appell functions F_1, F_2, F_3, F_4, Lauricella $F_B^{(N)}$, $F_D^{(N)}$, Lauricella-Saran F_S, F_T [3], Humbert Φ_1, Φ_2, Φ_3, Ξ_1, Ξ_2, into the BCF of a different structure, recurrence relations for these functions were used.

The report plans to consider the question of the correspondence of the constructed expansions into BCF to multiple power series.

1. W. B. Jones and W. J. Thron, *Continued Fractions: Analytic Theory and Applications*, Cambridge University Press, 1984.
2. L. Lorentzen and H. Waadeland, *Continued fractions with application*, Amsterdam: Elsevier Publisher B. V., 1992.
3. H. Exton, *Multiple hypergeometric functions and applications*, New York-Sydney-Toronto, Chichester, Ellis Horwood, 1976.

On decomposition in the Paley-Wiener space

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The decomposition problem of function from complete spaces into the product or sum of two functions from the "simple" classes was considered by many scientists, such as R.S. Yulmukhametov, Yu.I. Lyubarskii, I.E. Chyzhykov.

Let $E^p[\mathbb{C}(\alpha;\beta)], 0 < \beta - \alpha < 2\pi, 1 \le p < +\infty$, be the space of analytic in $\mathbb{C}(\alpha;\beta) = \{z : \alpha < \arg z < \beta\}$ functions f satisfying

$$\sup_{\alpha<\varphi<\beta} \Big\{ \int_0^{+\infty} |f(re^{i\varphi})| dr \Big\} < +\infty.$$

V.M. Dilnyi and T.I. Hishchak considered the following

Problem of decomposition. Does each function $f \in W_\sigma^1$ admit a decomposition $f = \chi + \mu$, where functions χ and μ are analytic in $\mathbb{C}_+ = \{z : \mathrm{Re} z > 0\}$ and $\chi \in E^1[\mathbb{C}(0; \frac{\pi}{2})]$, $\mu \in E^1[\mathbb{C}(-\frac{\pi}{2}; 0)]$?

Theorem Paley-Wiener. *Space W_σ^2 coincides with the space of functions represented as*

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_{-\sigma}^{\sigma} \varphi(it) e^{itz} dt, \quad \varphi \in L^2(-i\sigma; i\sigma).$$

We find a function χ of the form $\chi(z) = \chi_1(z) + i\chi_2(-iz)$, where

$$\chi_1(z) = \frac{1}{\sqrt{2\pi}} \int_0^{\sigma} \varphi(it) e^{itz} dt, \quad \chi_2(z) = -\frac{1}{\sqrt{2\pi}} \int_{-\sigma}^{0} \varphi(it) e^{itz} dt.$$

The function

$$\tilde{f}(z) = \frac{1}{\sqrt{2\pi}} \int_{-\sigma}^{\sigma} \tilde{\varphi}(it) e^{itz} dt, \quad \text{where} \quad \tilde{\varphi}(it) = \begin{cases} \varphi(it), & t > 0, \\ \varphi(-it), & t < 0, \end{cases}$$

is called *co-even* to $f(z)$.

Theorem 1. *Let $f \in W_\sigma^1$. Then for the function $\tilde{f}(z)$ the Problem of decomposition is solvable affirmatively.*

On the estimations for the distribution of holomorphic function in the unit disk

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We have got new estimations for the distribution of a holomorphic function in the unit disk that grows near various parts of the circle. We also get the corresponding estimation for Riesz's measure of subharmonic functions in the unit disk. These results generalize Favorov's and Golinskii's theorem on the subharmonic functions having the polynomial growth near a part of the boundary of the unit disk.

On some properties of local moduli of smoothness of conformal mapping

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Let a simply connected domain in the complex plane bounded by a smooth Jordan curve and a function realizing a homeomorphism of the closed unit disk onto the closure of this domain conformal in the open unit disk be given.

O. Kellog in 1912 proved that if the angle between the tangent to the curve and the positive real axis satisfies the Hölder condition, then the derivative of the function realizing conformal mapping satisfies the Hölder condition with the same index. Connection between properties of the boundary of the domain and properties of the considered function was investigated in works of numerous authors.

We consider results for local moduli of smoothness for derivatives of the mapping between the closures of two simply connected domains bounded by the smooth Jordan curves.

Regularity of growth of the logarithms of entire functions of improved regular growth in the metric of $L^p[0, 2\pi]$

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An entire function f is called a function of *improved regular growth* [1, 2, 3, 4] if for certain $\rho \in (0, +\infty)$ and $\rho_1 \in (0, \rho)$, and a 2π-periodic ρ-trigonometrically convex function $h(\varphi) \not\equiv -\infty$ there exists a set $U \subset \mathbb{C}$ contained in the union of disks with finite sum of radii and such that

$$\log |f(z)| = |z|^\rho h(\varphi) + o(|z|^{\rho_1}), \quad U \not\ni z = re^{i\varphi} \to \infty.$$

If an entire function f is a function of improved regular growth, then it has [1] the order ρ and the indicator $h(\varphi)$. Assume that the function $\log f(z) = \log |f(z)| + i \arg f(z) := \int_0^z f'(\zeta)/f(\zeta)\, d\zeta$, $\log f(0) = 0$, is defined in the complex plane with radial cuts from the zeros of the entire function f to ∞.

Let $\widetilde{h}(\varphi) = h(\varphi) - i\frac{h'(\varphi)}{\rho}$.

Theorem. *An entire function f of order $\rho \in (0, +\infty)$ with zeros on a finite system of rays $\{z : \arg z = \psi_j\}$, $j \in \{1, \dots, m\}$, $0 \le \psi_1 < \psi_2 < \ldots < \psi_m < 2\pi$, is a function of improved regular growth if and only if for certain $\rho_2 \in (0, \rho)$ and every $p \in [1, +\infty)$, one has*

$$\left\{ \frac{1}{2\pi} \int_0^{2\pi} \left| \frac{\log f(re^{i\varphi})}{r^\rho} - \widetilde{h}(\varphi) \right|^p d\varphi \right\}^{1/p} = o(r^{\rho_2 - \rho}), \quad r \to +\infty.$$

1. B. V. Vynnyts'kyi and R. V. Khats', *On growth regularity of entire function of noninteger order with zeros on a finite system of rays*, Mat. Stud. **24**:1 (2005), 31–38 (in Ukrainian).
2. R. V. Khats', *On entire functions of improved regular growth of integer order with zeros on a finite system of rays*, Mat. Stud. **26**:1 (2006), 17–24.
3. R. V. Khats', *Regularity of growth of Fourier coefficients of entire functions of improved regular growth*, Ukr. Math. J. **63**:12 (2012), 1953–1960.
4. R. V. Khats', *Asymptotic behavior of entire functions of improved regular growth in the metric of $L^p[0, 2\pi]$*, Carpathian Math. Publ. **5**:2 (2013), 341–344.

Extension property

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A subset V of a domain D in $\mathbb{C}^n$ is said to have extension property (EP-set) if any bounded holomorphic function on V can be extended to a bounded holomorphic function F on D such that $\sup_V |f| = \sup_D |F|$. EP set in the bidisc were charactarized by Agler and McCarthy in "Norm Preserving Extensions of Holomorphic Functions from Subvarieties of the Bidisk", Annals of Mathematics Vol. 157, No. 1, 2003.

The aim of the present talk is to study the problem in the tridisc. It is based on a joint paper with John McCarthy.

Subnormal independent random variables and Levy's phenomenon for entire functions

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For an entire function f of the form

$$f(z) = \sum\nolimits_{k=0}^{+\infty} a_k z^{n_k}, \quad n_k \in \mathbb{Z}_+ \tag{1}$$

we denote

$$\mathcal{K}(f, \mathcal{Z}) = \Big\{ f(z,t) = \sum\nolimits_{k=0}^{+\infty} a_k Z_k(t) z^{n_k} : t \in [0,1] \Big\},$$

where $\mathcal{Z} = (Z_k(t))$ is a sequence of complex-valued random variables. From one result ([1]) obtained for entire Dirichlet series it follows that under the condition

$$(\exists \Delta \in (0;+\infty))(\exists \rho \in [1/2;1])(\exists D > 0): \quad |n(t) - \Delta t^{\rho}| \le D \quad (t \ge t_0), \tag{2}$$

(here $n(t) = \sum_{n_k \le t} 1$ is the counting function of the sequence (n_k)), the inequality

$$M_f(r) \le \mu_f(r) \ln^{(2\rho-1)/2+\varepsilon} \mu_f(r),$$

holds for any $\varepsilon > 0$ and all $r \in [r_0(\varepsilon); +\infty)$ outside some set E_1 of finite logarithmic measure, i.e. $\int_{E_1} d\ln r < +\infty$.

Suppose that (X_n) is a sequence of real independent subnormal random variables, i.e., there exists $D > 0$ such that $(\forall k \in \mathbb{N})$ $(\forall \lambda_0 \in \mathbb{R})$: $\mathbf{E}(e^{\lambda_0 X_k}) \le e^{D\lambda_0^2}$. The class of such random variables is denoted by Ξ.

Theorem 1 ([2]). *Let $Z(\omega) = X(\omega) + iY(\omega)$, $X \in \Xi$, $Y \in \Xi$ and f be entire function of the form (1), where a sequence (n_k) satisfies condition (2). Then for every $\varepsilon > 0$ there exists a set $E(\varepsilon)$ of finite logarithmic measure such that for all $r \in (r_0(t), +\infty) \setminus E$ almost surely in $\mathcal{K}(f, \mathcal{Z})$ the inequality*

$$M_f(r,t) := \max\{|f(z,t)| \colon |z| = r\} \le \mu_f(r) \ln^{(2\rho-1)/4} \mu_f(r) (\ln\ln \mu_f(r))^{3/2+\delta}$$

holds.

In the case where $Z = (Z_n(t))$ is a multuplicative system such that $|Z_n(t)| \le 1$ a.s. Theorem 1 proved in [3]. Also in [3] we find following theorem.

Theorem 2 ([3]). *If a sequence (n_k) satisfies condition (2), a sequence of complex valued variables $\mathcal{Z} = (Z_k)$ such that $|Z_k(t)| \ge 1$ a.s. $(k \ge 0)$, then there exists an entire function f of the form (1) such that*

$$\lim_{r \to +\infty} \frac{M_f(r,t)}{\mu_f(r)(\ln \mu_f(r))^{(2\rho-1)/4}} = +\infty$$

a.s. in $\mathcal{K}(f, \mathcal{Z})$.

1. M. M. Sheremeta, *Wiman-Valiron's method for entire functions, represented by Dirichlet series*, Dokl. USSR Acad. Sci. **240**:5 (1978), 1036–1039 (in Russian).
2. A. Kuryliak, *Subnormal independent random variables and Levyв Ђ^{TM}s phenomenon for entire functions*, Mat. Stud. **47**:1 (2017), 10–19.
3. O. B. Skaskiv, *Random gap series and Wiman's inequality*, Mat. Stud. **30**:1 (2008), 101–106.

Quasi-elliptic functions

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Properties of quasi-elliptic functions (i. e. certain generalization of elliptic functions) are investigated. For this class of functions, analogues of $\wp$, ζ and σ Weierstrass functions are constructed. The differential equation for the generalized Weierstrass $\wp$ function is found. Relation between quasi-elliptic and p-loxodromic functions is obtained.

On hyperbolic-valued norm in bicomplex analysis

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In [1], there was introduced for the first time the notion of hyperbolic-valued modulus $|\cdot|_k$. This modulus is defined on the set of bicomplex numbers $\mathbb{BC}$ and takes values in the set of non-negative hyperbolic numbers $\mathbb{D}^+$. In [2] there were developed many analytical and geometrical facts on the ring of bicomplex numbers and it was evident that the whole theory became quite rich when the hyperbolic-valued modulus $|\cdot|_k$ goes into action.

The notion of hyperbolic-valued modulus gave rise to the notion of hyperbolic-valued norm on bicomplex moduli, see [1]. In this talk I will present further developments on the bicomplex functional analysis where the hyperbolic-valued modulus plays a key role. Finally we will see how the set of hyperbolic numbers behaves inside the bicomplex numbers in a quite similar way as the set of real numbers behaves inside the complex numbers and will comment on some geometric consequences of this fact.

1. D. Alpay, M. E. Luna-Elizarrarás, M. Shapiro, and D.C. Struppa, *Basics of functional analysis with bicomplex scalars, and bicomplex Schur analysis*, Series Springer Briefs, Springer, (2014).
2. M. E. Luna-Elizarrarás, M. Shapiro, D. C. Struppa, and A. Vajiac, *Bicomplex holomorphic functions: the algebra, geometry and analysis of bicomplex numbers*, Series Frontiers in Mathematics, Birkhäuser, Springer International Publishing, Switzerland, (2015).

Interpolation in the space of functions of finite order in a half-plane

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The problem of multiple interpolation in the class of functions of finite order in the upper half-plane of the complex variable is considered. The problem belongs to the class of problems of free interpolation which considered by A.F. Leont'ev for the first time. Necessary and sufficient conditions of solvability of this problem are found. The found criteria are formulated in terms of the canonical products constructed on knots of interpolation, and in terms of the Nevanlinna measure determined by these knots. Work is a continuation of research of the first author considering similar problems in classes of analytic functions in the upper half-plane of finite order.

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The functions of completely regular growth in the half-plane

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The class $J\delta(\gamma(r))^o$ of delta-subharmonic functions of completely regular growth in the upper half-plane with respect to the function of growth γ is considered. The theory of the functions of completely regular growth (c.r.g.) in the half-plane are generalized, as well as in works by A.A. Kondratyuk, in two directions: 1) the growth of functions is measured with respect to an arbitrary growth function $\gamma(r)$, satisfying the condition $\gamma(2r) \leq M\gamma(r)$, for some $M > 0$ and all $r > 0$ only; 2) classes of delta-subharmonic functions of a c.r.g. in the half-plane are introduced.

The concept of the indicator of function $v \in J\delta(\gamma(r))^o$ is entered and some its properties are studied.

Paper reference: (**Dokl. RAN, 380:3 (2001), pp. 1–3.**).

On equicontinuity of some class of mappings, which are quasiconformal in the mean

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Given E and $F \subset \overline{\mathbb{R}^n}$, denote $\Gamma(E,F,D)$ the family of all curves $\gamma : [a,b] \to \overline{\mathbb{R}^n}$ joining E and F in D. In what follows, h is a chordal metric. Let $Q(x,t) = \{y \in \overline{\mathbb{R}^n} : h(x,y) < t\}$ be the spherical ball centered at x of radius t. Let $c(E,x) = \max\{m(E,x), m(E,\widetilde{x})\}$, where $c(E) = \inf\limits_{x\in\mathbb{R}^n} c(E,x)$, $m_t(E,r,x) = M(\Gamma(\partial Q(x,t), E \cap \overline{Q(x,r)}))$, $m(E,x) = m_{\sqrt{3}/2}(E, \frac{\sqrt{2}}{2}, x)$. Let $\Phi : [0,\infty] \to [0,\infty]$ be a non-decreasing convex function. Denote $\mathfrak{R}^{\Phi}_{M,\Delta}$ the family of all open discrete ring Q-mappings in D with $c\left(\overline{\mathbb{R}^n} \setminus f(D)\right) \geq \Delta$, and

$$\int\limits_D \Phi\left(Q(x)\right) \frac{dm(x)}{\left(1+|x|^2\right)^n} \ \leq \ M\,.$$

Theorem. *Let $\Phi : [0,\infty] \to [0,\infty]$ be a non-decreasing convex function. If*

$$\int\limits_{\delta_0}^{\infty} \frac{d\tau}{\tau\left[\Phi^{-1}(\tau)\right]^{\frac{1}{n-1}}} \ = \ \infty$$

for some $\delta_0 > \tau_0 := \Phi(0)$, then $\mathfrak{R}^{\Phi}_{M,\Delta}$ is equicontinuous, and, consequently, forms a normal family of mappings for all $M \in (0,\infty)$ and $\Delta \in (0,1)$.

Let $\mathfrak{R}_{Q,\Delta}(D)$ be a class of all open discrete ring Q-mappings f in a domain $D \subset \mathbb{R}^n$, $n \geq 2$, with $c(\overline{\mathbb{R}}^n \setminus f(D)) \geq \Delta{>}0$. To prove the theorem mentioned above, we have established the following assertion.

Lemma. *Let $\Delta > 0$ and let $Q : D \to [0,\infty]$ be a measurable function. Now*

$$h\left(f(x), f(x_0)\right) \leq \frac{\omega_{n-1}}{c_n\Delta} \cdot \Big(\int\limits_{|x-x_0|}^{\varepsilon(x_0)} \frac{dr}{r q_{x_0}^{\frac{1}{n-1}}(r)} \Big)^{-1}$$

for every $f \in \mathfrak{R}_{Q,\Delta}(D)$ and $x \in B(x_0, \varepsilon(x_0))$, $\varepsilon(x_0){<}\operatorname{dist}(x_0, \partial D)$, where $c_n{>}0$ depends only on n, and $q_{x_0}(r)$ is a mean integral value of the function $Q(z)$ on $|z-x_0| = r$.

Paper reference: (**http://arxiv.org/abs/1607.00677**).

Sufficient conditions for existence of angular υ-density of zeros for entire functions in terms of characteristics of its logarithmic derivative

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Let f be an entire function of zero order, $f(0) = 1$; $n(r, \alpha, \beta)$ be the counting function for zeros of f in the sector $\{z\colon |z| \leq r, \alpha \leq \arg z < \beta\}$, $n(r) = n(r, 0, 2\pi)$. By L we denote the class of continuously differentiable on $\mathbb{R}_+$ growth functions υ such that $r\upsilon'(r)/\upsilon(r) \to 0$ as $r \to +\infty$; $H_0(\upsilon)$, $\upsilon \in L$, the class of entire functions f of zero order such that $0 < \Delta = \varlimsup\limits_{r\to+\infty} n(r)/\upsilon(r) < +\infty$.

We say that zeros of $f \in H_0(\upsilon)$ have an angular υ-density, $\upsilon \in L$, if the limit

$$\Delta(\alpha, \beta) = \lim_{r\to+\infty} n(r, \alpha, \beta)/\upsilon(r)$$

exists for all $0 \leq \alpha < \beta < 2\pi$ with the exception of at most a countable number of values $0 \leq \alpha < \beta < 2\pi$. Let $F(z) = zf'(z)/f(z)$ be the logarithmic derivative of f with respect to $\log z$, $c_k(r, F) = 1/(2\pi) \int\limits_0^{2\pi} F(re^{i\varphi})e^{-ik\varphi} d\varphi$, $k \in \mathbb{Z}$, be its Fourier coefficients.

The necessary conditions for the existence of the angular υ-density of zeros of $f \in H_0(\upsilon)$ in terms of the regular growth of $c_k(r, F)$ and F in $L^p[0, 2\pi]$-metric were found in [1]. In generally the converse statements are false. Because of that we consider some corrected relations for kth Fourier coefficients $c_k(r, F)$ and the convergence of F in $L^p[0, 2\pi]$-metric and show that under these conditions zeros of f have the angular υ-density.

1. M. V. Zabolotskyi and M. R. Mostova, *Logarithmic derivative and the angular density of zeros for a zero-order entire function*, Ukr. Math. J., **66**:4 (2014), 530–540.

On the entire functions from the Laguerre-Pólya class having the decreasing second quotient of Taylor coefficients

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We consider entire functions with positive Taylor coefficients and investigate the conditions under which these functions belong to the Laguerre-Pólya class.

Let $f(z) = \sum_{k=0}^{\infty} a_k z^k, a_k > 0$, be an entire function. We suppose that the quotients $\frac{a_{n-1}^2}{a_{n-2}a_n}$ are decreasing in n. Let $b := \lim\limits_{n\to\infty} \frac{a_{n-1}^2}{a_{n-2}a_n}$. We have found the minimal constant q_∞ ($q_\infty \approx 3.2336$) such that if $b \geq q_\infty$, then all the zeros of f are real and negative (in other words, such functions belong to the Laguerre-Pólya class).

Dominating polynomial of power series of analytic in a bidisc functions of bounded **L**-index in joint variables

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We use definitions and notations from [1, 2]. We develop an analytic in $\mathbb{D}^2$ function $F(z)$ in the power series in a neighborhood of point $z^0 \in \mathbb{D}^2$ in the diagonal form

$$F(z) = \sum_{k=0}^{+\infty} p_k(z_1 - z_1^0, z_2 - z_2^0), \tag{1}$$

where $p_k(\tau_1, \tau_2) = \sum_{j_1+j_2=k} b_{j_1,j_2} \tau_1^{j_1} \tau_2^{j_2}$ are homogeneous polynomials of degree k. The polynomial $p_{k_0}, k_0 \in \mathbb{Z}_+$, is called *a dominating polynomial* in the power series expansion (1) on $\mathbb{T}^2(z^0, R) \subset \mathbb{D}^2$ if for every $z \in \mathbb{T}^2(z^0, R)$ the next inequality holds:

$$\left| \sum_{k \neq k_0} p_k(z_1 - z_1^0, z_2 - z_2^0) \right| \leq \frac{1}{2} \max\{|b_{j_1,j_2}| r_1^{j_1} r_2^{j_2} : j_1 + j_2 = k_0\},$$

where $b_{j_1,j_2} = \frac{F^{(j_1,j_2)}(z^0)}{j_1! j_2!}$, $R = (r_1, r_2)$, $|z_1 - z_1^0| = r_1$, $|z_2 - z_2^0| = r_2$.

Theorem 1. *Let $\beta > 1$, $\mathbf{L} \in Q(\mathbb{D}^2)$. If an analytic function F in $\mathbb{D}^2$ has bounded $\mathbf{L}$-index in joint variables then there exists $p \in \mathbb{Z}_+$ that for all $d \in (0;\beta]$ there exists $\eta(d) \in (0;d)$ such that for each $z^0 \in \mathbb{D}^2$ and some $r = r(d, z^0) \in (\eta(d), d)$, $k_0 = k_0(d, z^0) \le p$ the polynomial p_{k_0} is the dominating polynomial in the series (1) on $\mathbb{T}^2(z^0, \frac{R}{\mathbf{L}(z^0)})$ with $R = (r, r)$.*

Theorem 2. *Let $\beta > 1$, $\mathbf{L} \in Q(\mathbb{D}^2)$. If there exist $p \in \mathbb{Z}_+$, $d \in (0;1]$, $\eta \in (0;d)$ such that for each $z^0 \in \mathbb{D}^2$ and some $R = (r_1, r_2)$ with $r_j = r_j(d, z^0) \in (\eta, d)$, $j \in \{1, 2\}$, and certain $k_0 = k_0(d, z^0) \le p$ the polynomial p_{k_0} is a dominating polynomial in the series (1) on $\mathbb{T}^2(z^0, R/\mathbf{L}(z^0))$ then the analytic in $\mathbb{D}^2$ function F has bounded $\mathbf{L}$-index in joint variables.*

1. A. I. Bandura and N. V. Petrechko, *Properties of power series of analytic in a bidisc functions of bounded* **L***-index in joint variables*, Carpathian Math. Publ. **9**:1 (2017), 6–12;
   doi:10.15330/cmp.9.1.6-12
2. A. I. Bandura and O. B. Skaskiv, *Some properties of analytic in a ball functions of bounded L-index in joint variables*, arXiv:1705.09568v2[math.CV], 5 Jun 2017, 55 p.

Paper reference:
(**http://arxiv.org/abs/1609.04190**, **doi:10.15330/cmp.9.1.6-12**).

Removability results for subharmonic functions, for harmonic functions and for holomorphic functions

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We begin with an improvement to an extension result for subharmonic functions of Blanchet et al. With the aid of this improvement we then give extension results both for harmonic and for holomorphic functions. Our results for holomorphic functions are related to Besicovitch's and Shiffman's well-known extension results, at least in some sense. Moreover, we recall another, slightly related and previous extension result for holomorphic functions.

Paper reference: (**http://arxiv.org/abs/1607.07029v3**).

Entire Dirichlet series and h-measure of exceptional sets

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Let $D(\Lambda)$ be the class of entire Dirichlet series of the form

$$F(z)=\sum_{n=0}^{+\infty} a_n e^{z\lambda_n},$$

where $\Lambda=(\lambda_n)$, $0=\lambda_0<\lambda_n\uparrow+\infty$ $(1\le n\to+\infty)$. For $F\in D(\Lambda)$ and $x\in\mathbb{R}$ we denote

$$M(x,F)=\sup\{|F(x+iy)|\colon y\in\mathbb{R}\},$$
$$m(x,F)=\inf\{|F(x+iy)|\colon y\in\mathbb{R}\},$$
$$\mu(x,F)=\max\{|a_n|e^{x\lambda_n}\colon n\ge 0\}.$$

It is known [1] that for every entire function $F\in D(\Lambda)$ the relations (A):

$$M(x,F)\sim\mu(x,F),\qquad M(x,F)\sim m(x,F)$$

hold as $x\to+\infty$ outside some exceptional set E of finite Lebesgue measure, if and only if

$$\sum_{n=0}^{+\infty}\frac{1}{\lambda_{n+1}-\lambda_n}<+\infty.$$

The finiteness of the Lebesgue measure of an exceptional set E is the best possible description of an exceptional set E in the class $D(\Lambda)$ ([2]).

Let Φ be a positive increasing to $+\infty$ continuous function on $[0;+\infty)$, φ be the inverse function to Φ and h be a positive differentiable function on $[0;+\infty)$. Denote

$$D(\Lambda,\Phi)=\{F\in D(\Lambda):\ (\exists C>0)[\ln\mu(x,F)\ge Cx\Phi(Cx)\ (x>x_0)\},$$
$$D_\varphi(\Lambda):=\big\{F\in D(\Lambda)\colon\ (\exists n_0)(\forall n\ge n_0)[|a_n|\le\exp\{-\lambda_n\varphi(\lambda_n)\}]\big\}.$$

Theorem [3]. *If $h'(x)$ is non-decreasing to $+\infty$ and*

$$(\forall b>0):\quad \sum_{k=0}^{+\infty}\frac{h'\big(b\varphi(b\lambda_k)\big)}{\lambda_{k+1}-\lambda_k}<+\infty,$$

then for all $F\in D(\Lambda,\Phi)$ relations (A) hold as $x\to+\infty$ outside some set E of finite h-measure ($\int_E dh(x)<+\infty$) uniformly in $y\in\mathbb{R}$.

Conjecture. If $h'(x)$ is non-increasing and

$$\sum_{n=0}^{+\infty}\frac{h'(\varphi(\lambda_n))}{\lambda_{n+1}-\lambda_n}<+\infty,$$

then for all $F\in D_\varphi(\Lambda)$ relations (A) hold as $x\to+\infty$ outside some set E of finite h-measure uniformly in $y\in\mathbb{R}$.

1. O. B. Skaskiv, *Maximum of the modulus and maximal term of an integral Dirichlet series*, Dop. Akad. Nauk Ukr. SSR, Ser. A **11** (1984), 22–24.
2. T. M. Salo, O. B. Skaskiv, and O. M. Trakalo, *On the best possible description of exceptional set in Wiman–Valiron theory for entire functions*, Mat. Stud. **16**:2 (2001), 131–140.
3. T. M. Salo and O. B. Skaskiv, *The minimum modulus of gap power series and h-measure of exceptional sets*, arXiv: 1512.05557v1 [math.CV] 17 Dec 2015, 13 p.

Best approximation of the Cauchy–Szegő kernel in the mean on the unit circle

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Let $\mathbf{a} := \{a_k\}_{k=0}^{\infty}$ be a sequence of points in the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ among which there may be points of finite or ever infinite multiplicity. A system $\varphi := \{\varphi_j\}_{j=0}^{\infty}$ of functions φ_j of the form

$$\varphi_0(t) = \frac{\sqrt{1-|a_0|^2}}{1-\overline{a_0}t}, \quad \varphi_j(t) = \frac{\sqrt{1-|a_j|^2}}{1-\overline{a_j}t}B_j(t),\ j = 1, 2, \ldots,$$

where

$$B_j(t) := \prod_{k=0}^{j-1} \frac{-|a_k|}{a_k} \cdot \frac{t-a_k}{1-t\overline{a_k}}, \quad \frac{|a_k|}{a_k} = -1 \text{ for } a_k = 0,$$

is called a Takenaka–Malmquist system.

By TM we denote the set of all Takeneka–Malmquist systems.

It is known that for arbitrary $\varphi \in TM$ and $n \in \mathbb{N}$,

$$\frac{1}{1-\overline{z}t} - \sum_{j=0}^{n-1} \overline{\varphi_j(z)}\varphi_j(t) = \frac{\overline{B_n(z)}B_n(t)}{1-\overline{z}t}, \quad (z,t) \in \overline{\mathbb{D}}^2 \setminus \mathbb{T}^2, \tag{1}$$

where $\mathbb{T} := \{t \in \mathbb{C} : |t| = 1\}$.

It follows from (1) that

$$\min_{\lambda_{j,n}(z)} \int_{\mathbb{T}} \Big| \frac{1}{1-\overline{z}t} - \sum_{j=0}^{n-1} \lambda_{j,n}(z)\varphi_j(t) \Big|^2 d\sigma(t) =$$

$$= \int_{\mathbb{T}} \Big| \frac{1}{1-\overline{z}t} - \sum_{j=0}^{n-1} \overline{\varphi_j(z)}\varphi_j(t) \Big|^2 d\sigma(t) = \frac{|B_n(z)|^2}{1-|z|^2}, \qquad z \in \mathbb{D},$$

where σ is the normalized Lebesgue measure on the circle $\mathbb{T}$.

We compute the values

$$E_n\Big(\frac{1}{1-\overline{z}\cdot}\ ; \varphi\Big) := \min_{\lambda_{j,n}(z)} \int_{\mathbb{T}} \Big| \frac{1}{1-\overline{z}t} - \sum_{j=0}^{n-1} \lambda_{j,n}(z)\varphi_j(t) \Big| d\sigma(t), \tag{2}$$

which is called the best approximations in the mean on the unit circle $\mathbb{T}$ for the Cauchy–Szegő kernel $1/(1-z\bar{t})$ by quasipolynomials with respect to the Takenaka–Malmquist system φ.

Theorem. *Let $\varphi \in TM$ and let $z \in \mathbb{D}$. Then, for every $n \in \mathbb{N}$,*

$$E_n\Big(\frac{1}{1-\overline{z}\cdot}\ ;\varphi\Big) = |B_n(z)|\frac{1-|z|^2}{1-|zB_n(z)|^2}\int_{\mathbb{T}}\frac{|1-\overline{zB_n(z)}tB_n(t)|}{|1-\overline{z}t|^2}\,d\sigma(t). \qquad (3)$$

The minimum in (2) is attained for the coefficients

$$\lambda^*_{j,n}(z) = \frac{\overline{\varphi_j(z)}}{1-|zB_n(z)|^2}\left(1-z\frac{1-a_j\overline{z}}{z-a_j}\left|\frac{B_n(z)}{B_j(z)}\right|^2\right),\ j=0,1,\ldots,n-1.$$

In the case $a_k = 0, k = 0,1,\ldots$, the equality (3) for the first time were obtained by Alper (1963).

On boundary behavior of ring Q-mappings in terms of prime ends

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By definition, a chain of cross-cuts $\{\sigma_m\}$ determines a *chain of domains* $d_m \subset D$ such that $\partial\, d_m \cap D \subset \sigma_m$ and $d_1 \supset d_2 \supset \cdots \supset d_m \supset \cdots$. Two chains of cross-cuts $\{\sigma_m\}$ and $\{\sigma'_k\}$ are said to be equivalent if for every $m \in \mathbb{N}$ the domain d_m contains all domains d'_k except finitely many of them, and for every $k \in \mathbb{N}$ the domain d'_k also contains all domains d_m except finitely many. An *end* of a domain D is an equivalence class of chains of cross-cuts of D. We say that an end K is a *prime end* if K contains a chain of cross-cuts $\{\sigma_m\}$, such that $\lim\limits_{m\to\infty} M(\Gamma(C,\sigma_m,D)) = 0$ for some continuum C in D, where M is the modulus of the family $\Gamma(C,\sigma_m,D)$.

We say that the boundary of a domain D in $\mathbb{R}^n$ is *locally quasiconformal* if every point $x_0 \in \partial D$ has a neighborhood U that admits a conformal mapping φ onto the unit ball $\mathbb{B}^n \subset \mathbb{R}^n$ such that $\varphi(\partial D \cap U)$ is the intersection of $\mathbb{B}^n$ and a coordinate hyperplane. We say that a bounded domain D in $\mathbb{R}^n$ is *regular* if D can be mapped quasiconformally onto a domain with a locally quasiconformal boundary. If $\overline{D}_P$ is the completion of a regular domain D by its prime ends and g_0 is a quasiconformal mapping of a domain D_0 with locally quasiconformal boundary onto D, then this mapping naturally determines the metric $\rho_0(p_1,p_2) = |\tilde{g_0}^{-1}(p_1) - \tilde{g_0}^{-1}(p_2)|$, where $\tilde{g_0}$ is the extension of g_0 onto $\overline{D_0}$. Let $x = (x_1,\ldots,x_n)$ and $f(x) = (f_1(x),\ldots,f_n(x))$. Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, and $f : D \to \mathbb{R}^n$ be a continuous mapping. A mapping $f : D \to \mathbb{R}^n$ is said to be *discrete* if the preimage $f^{-1}(y)$ of every point $y \in \mathbb{R}^n$ consists of isolated points, and *open* if the image of every open set $U \subset D$ is open in $\mathbb{R}^n$. A mapping f is closed if the image of every closed set $U \subset D$ is closed in

$f(D)$. Given a domain D and two sets E and F in $\overline{\mathbb{R}^n}$, $n \geq 2$, $\Gamma(E, F, D)$ denotes the family of all paths $\gamma : [a, b] \to \overline{\mathbb{R}^n}$ which join E and F in D, i.e., $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in D$ for $a<t<b$. Denote

$$S(x_0, r_i) = \{x \in \mathbb{R}^n : |x - x_0| = r_i\}, \quad i = 1, 2,$$

$$A(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1<|x - x_0|<r_2\}.$$

Given a (Lebesgue) measurable function $Q : D \to [0, \infty]$ with $Q(x) = 0$ for $x \notin D$, a mapping $f : D \to \mathbb{R}^n$ is called *ring Q-mapping at a point $x_0 \in \overline{D}$* if

$$M(f(\Gamma(S_1, S_2, D))) \leqslant \int\limits_{A(x_0,r_1,r_2)\cap D} Q(x) \cdot \eta^n(|x - x_0|)\, dm(x)$$

for any $A(x_0, r_1, r_2)$, $0<r_1<r_2<r_0$, some $r_0>0$ and for every Lebesgue measurable function $\eta : (r_1, r_2) \to [0, \infty]$ with $\int\limits_{r_1}^{r_2} \eta(r)dr \geqslant 1$. The following results hold.

Theorem 1. *Let $n \geqslant 2$, let D be a regular domain in $\mathbb{R}^n$, and let D' be a bounded domain with locally quasiconformal boundary. Assume that $f : D \to D'$ is open, discrete and closed ring Q-mapping at ∂D, $D' = f(D)$. Then f has a continuous extension $f : \overline{D}_P \to \overline{D'}_P$ such that $f(\overline{D}_P) = \overline{D'}_P$, whenever*

$$\int\limits_{\varepsilon}^{\varepsilon_0} \frac{dt}{tq_{x_0}^{\frac{1}{n-1}}(t)} < \infty\,, \qquad \int\limits_{0}^{\varepsilon_0} \frac{dt}{tq_{x_0}^{\frac{1}{n-1}}(t)} = \infty \tag{1}$$

for every $x_0 \in \partial D$, where

$$q_{x_0}(r) := \frac{1}{\omega_{n-1}r^{n-1}} \int\limits_{|x-x_0|=r} Q(x)\, d\mathcal{H}^{n-1}.$$

Theorem 2. *The statement of Theorem 1 is true, if instead of the conditions (1) we require that $Q \in FMO(x_0)$ for every $x_0 \in \partial D$.*

Paper reference: (**arXiv:1602.00660**).

On a multiple interpolation problem in a class of entire functions with the fast-growing interpolation knots

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Let $(\lambda_k : k \in \mathrm{N})$ be a sequence of distinct nonzero complex numbers, which have no finite limit points, $N_\lambda(r) = \sum\limits_{|\lambda_k|\le r} \log \frac{r}{|\lambda_k|}$, g be an entire function and

$$M_g(r) = \max\{|g(z)| : |z| = r\}.$$

Various interpolation problems in the classes of entire functions were investigated in the works of many authors. A. Gel'fond [1] and Y. Kaz'min [2] considered an interpolation problem $g(\lambda_k) = b_k$, $k \in \mathrm{N}$, with interpolation knots in $\lambda_k = q^{k-1}$, $|q| > 1$. From their results the next theorems follow.

Theorem K (Kaz'min)**.** *Let $\lambda_k = q^{k-1}, |q| > 1$. Then for every sequence (b_k),*

$$\overline{\lim_{k\to\infty}} |q|^{-\frac{(k-1)}{2}} |b_k|^{1/k} \le r_1, \quad r_1 \in (\Delta; 1),$$

interpolation problem $g(\lambda_k) = b_k$ has a unique solution in the class of entire functions g, that satisfy the condition

$$\ln M_g(r) \le \frac{\ln^2 \rho_1 r}{2\ln|q|} + \frac{\ln r}{2} + c_2$$

for each $\rho_1 > r_1$.

The aim of this paper is to consider the interpolation problem

$$g(\lambda_k) = b_{k,1}, \quad g'(\lambda_k) = b_{k,2}, \quad k \in \mathbb{N}, \tag{1}$$

in the case, when for some $\Delta \in (0; 1)$ the sequence (λ_k) satisfies the condition

$$|\lambda_k/\lambda_{k+1}| \le \Delta, \quad k \in \mathbb{N}. \tag{2}$$

Let $L(z) = \prod\limits_{j=1}^{\infty} \left(1 - \frac{z}{\lambda_j}\right)$. Our main result is the following.

Theorem 1. *Let (λ_k) be a sequence of complex numbers satisfies condition (2) for some $\Delta < 1$. Then for every sequences $(b_{n,1})$ and $(b_{n,2})$ such that for some $q \in (\Delta; 1)$ (here and further c_i is a positive constant),*

$$|b_{k,1}| \le c_1 \exp\left(2N_\lambda\left(q\,|\lambda_k|\right)\right), \; |\lambda_k|\,|b_{k,2}| \le c_5 \exp\left(N_\lambda\left(|\lambda_k|\right) + N_\lambda(q\,|\lambda_k|\right), k \in \mathbb{N},$$

interpolation problem (1) has the unique solution in the class of entire functions g that satisfy the condition

$$M_g(r) \le c_3 \exp\left(N_\lambda(r) + N_\lambda(\rho_1 r)\right)$$

for each $\rho_1 \in (q;1)$. *In so doing, the function*

$$g(z)=\sum_{k=1}^{\infty}\left(-\frac{L''(\lambda_k)}{L'^3(\lambda_k)}b_{k,1}\frac{L^2(z)}{z-\lambda_k}+\frac{1}{L'^2(\lambda_k)}b_{k,2}\frac{L^2(z)}{z-\lambda_k}+\frac{1}{L'^2(\lambda_k)}b_{k,1}\frac{L^2(z)}{(z-\lambda_k)^2}\right)$$

is this solution.

1. A. O. Gel'fond, *Claculation of finite differences*, Moscow, 1967.
2. Yu. A. Kaz'min, *On a problem of A. O. Gel'fond*, Math. USSR-Sb. **19**:4 (1973), 509–530.
3. B. V. Vynnyts'kyi and I. B. Sheparovych, *On the existence of an interpolation problem solution in one class of entire functions, which is determined by a counting function*, Actual Probl. Phys. Math. Comput. Sc. **4** (2012), 12–14.

Old and new in fixed point theory towards complex dynamics

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Historically, Complex Dynamics and Geometric Function Theory have been intensively developed from the beginning of the twentieth century. They provide the foundations for broad areas of Mathematics. In the last fifty years the theory of holomorphic and pseudo-contractive mappings on complex spaces has been studied by many mathematicians with many applications to nonlinear analysis, functional analysis, differential equations, classical and quantum mechanics. The laws of dynamics are usually presented as equations of motion which are written in the abstract form of a dynamical system: $\frac{dx}{dt}+f(x)=0$, where x is a variable describing the state of the system under study, and f is a vector-function of x. The study of such systems when f is a monotone (or an accretive, generally nonlinear) operator on the underlying space has been recently the subject of much research by analysts working on quite a variety of interesting topics, including boundary value problems, integral equations and evolution problems.

There is a long history associated with the problem on iterating holomorphic mappings and their fixed points, the modern work of M. Abate, K. Goebel, T. Kuczumow, S. Reich, J.-P. Vigue being among very important.

In this talk we give a brief description of the classical statements which combine celebrated Julia's Theorem in 1920, Carathéodory's contribution in 1929 and Wolff's boundary version of the Schwarz Lemma in 1926 and their modern interpretations for discrete and continuous semigroups of hyperbolically nonexpansive mappings in Hilbert and Banach spaces. Also we present flow-invariance conditions for holomorphic and hyperbolically monotone mappings in the spirit of F. Brauder, W. A. Kirk, S. Reich and resolvent methods developed by them.

Finally we give some applications of complex dynamical systems to geometry of domains in complex spaces and operator theory.

Linearly convex functions in a hypercomplex space

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We shall consider the n-dimensional hypercomplex space $\mathbb{H}^n$, $n \in \mathbb{N}$, which is a direct product of n copies of the ring of quaternions $\mathbb{H}$ ($\mathbb{H}^1 := \mathbb{H}$).

A function $f\colon \mathbb{H}^n \multimap \mathbb{H}$ is called *multivalued*, if the image of the point $x \in \mathbb{H}^n$ is a subset $f(x) \subset \mathbb{H}$.

A multivalued function $f\colon \mathbb{H} \multimap \mathbb{H}$ is called *linearly convex*, if for any pair of points $(x_0, y_0) \in \mathbb{H}^{n+1} \setminus \Gamma(f)$ there is an affine function l such that $y_0 = l(x_0)$ and $l(x) \cap f(x) = \emptyset$ for all of $x \in \mathbb{H}^n$, where $\Gamma(f)$ stands for the grah of the function f.

The *function conjugated* with f is the function given by the equality

$$f^*(y) = \mathbb{H}^o \setminus \bigcup_x (\langle x, y\rangle - f(x)).$$

We find a function conjugate to the function $f^*(x)$.

Theorem 1. *For every function $f\colon \mathbb{H}^n \multimap \mathbb{H}$ the inclusion $f \subset f^{**}$ holds.*

The next theorem is a hypercomplex analogue of the Fennel-Moro theorem.

Theorem 2. *Let $f\colon \mathbb{H}^n \multimap \mathbb{H}$ be a multivalued function such that $\mathbb{H} \setminus f(x) \neq \emptyset$ for all $x \in \mathbb{H}^n$. Then $f^{**} = f$ is if and only if f is linearly convex.*

The abscissa of absolute convergence of Dirichlet series with random exponents

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Let $(\Omega, \mathcal{A}, P)$ be a probability space, $\Lambda = \big(\lambda_k(\omega)\big)_{k=0}^{+\infty}$ be a sequence of positive random variables on it. Let $\mathcal{D}(\Lambda)$ be the class of formal random Dirichlet series of the form

$$f_\omega(z) = f(z, \omega) = \sum\nolimits_{k=0}^{+\infty} f_k e^{z\lambda_k(\omega)} \quad (z \in \mathbb{C},\ \omega \in \Omega),\ f_k \in \mathbb{C}.$$

We assume, that the condition $\ln k = o(\ln |f_k|) \quad (k \to +\infty)$ holds. Let $\sigma(\omega) := \sigma(f, \omega)$ be the abscissa of absolute convergence of this series for fixed $\omega \in \Omega$.

Theorem 1. *Let $f \in \mathcal{D}(\Lambda)$ and $\Lambda = \big(\lambda_k(\omega)\big)$ be a sequence of pairwise independent random variables with distribution functions $F_k(x) := P\{\omega : \lambda_k(\omega) < x\}$, $x \in \mathbb{R}$, $k \geq 0$. The following assertions hold:*

i) *If* $\sigma(\omega) \geq \rho \in (0, +\infty)$ *a.s. then* $\forall \varepsilon \in (0, \rho))$:

$$\sum_{k=0}^{+\infty} \big(1 - F_k(\ln|f_k|/(-\rho+\varepsilon))\big) < \infty.$$

ii) *If* $0 \geq \sigma(\omega) \geq \rho \in (-\infty, 0]$ *a.s. then*

$$(\forall \varepsilon > 0) \colon \sum_{k=0}^{+\infty} F_k(\ln|f_k|/(-\rho+\varepsilon)) < \infty.$$

Theorem 2 ([1, 2])**.** *Let* $\Lambda = \big(\lambda_k(\omega)\big)$ *be a sequence of random variables with distribution functions* $F_k(x) := P\{\ \omega : \lambda_k(\omega) < x\}$, $x \in \mathbb{R}$, $k \geq 0$, *and* $f \in \mathcal{D}(\Lambda)$. *The following assertions hold:*

i) *If there exist* $\rho \in (0, +\infty)$ *and a sequence* (ε_k) *such that* $\varepsilon_k \to +0$ $(k \to +\infty)$ *and* $\sum_{k=0}^{+\infty} \big(1 - F_k(\frac{\ln|f_k|}{-\rho+\varepsilon_k})\big) < +\infty$, *then* $\sigma(\omega) \geq \rho$ *a.s.*

ii) *If there exist* $\rho \in (-\infty, 0]$ *and a sequence* (ε_k) *such that* $\varepsilon_k \to +0$ $(k \to +\infty)$ *and* $\sum_{k=0}^{+\infty} F_k(\frac{\ln|f_k|}{-\rho+\varepsilon_k}) < +\infty$, *then* $\sigma(\omega) \geq \rho$ *a.s.*

Corollary 1. *Let* $f \in \mathcal{D}(\Lambda)$ *and* $\Lambda = (\lambda_k(\omega))$ *be a sequence of pairwise independent random variables with distribution functions* $F_k(x)$, $k \geq 0$. *If* $\varliminf\limits_{k\to+\infty} F_k(+0) < 1$ *and* $f_k \to 0$ $(k \to +\infty)$, *then* $\sigma(f, \omega) = 0$ *a.s.*

Corollary 2. *Let* $f \in \mathcal{D}(\Lambda)$ *and* $\Lambda = (\lambda_k(\omega))$ *be a sequence of pairwise independent random variables with distribution functions* $F_k(x)$, $k \geq 0$. *If there exists a positive random variable* $a(\omega)$ *such that* $(\forall x \geq 0)(\forall k \in \mathbb{Z}_+)\colon$ $F_k(x) \leq F_a(x) := P\{\omega\colon a(\omega) < x\}$ *and* $F_a(+0) < 1$ *and* $f_k \to 0$ $(k \to +\infty)$, *then* $\sigma(f, \omega) = 0$ *a.s.*

1. A. O. Kuryliak, O. B. Skaskiv, and N. Yu. Stasiv, *On the abscissas of convergence of Dirichlet series with random pairwise independent exponents*, ArXiv:1703.03280v1 [math.CV] 12 Mar 2017, 12 p.
2. A. O. Kuryliak, O. B. Skaskiv, and N. Yu. Stasiv, *On the convergence of Dirichlet series with random exponents*, Int. J. Appl. Math., **30**:3 (2017), 229–238.

On the convergence classes for analytic in a ball functions

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Let $\mathcal{A}^p_R$ be the class of analytic functions f in $B_R = \{z \in \mathbb{C}^p \colon |z| < R\}$, represented by the power series

$$f(z) = f(z_1, \dots, z_p) = \sum_{\|n\|=0}^{+\infty} a_n z^n,$$

where $0 < R \leq +\infty$, $z^n = z_1^{n_1} \dots z_p^{n_p}$, $p \in \mathbb{N}$, $p \geq 2$, $n = (n_1, \dots, n_p) \in \mathbb{Z}^p_+$, $\|n\| = \sum_{j=1}^p n_j$. For $r < R$ and a function $f \in \mathcal{A}^p_R$ we denote $M_f(r) = \max\{|f(z)| \colon |z| \leq r\}$.

Theorem 1. *Let $f \in \mathcal{A}^p_1$ and $A_k = \max\{|a_n| : \|n\| = k\}$, $k \in \mathbb{Z}_+$.*

1. *If $A_k/A_{k+1} \nearrow 1$ $(k \uparrow +\infty)$ and $\sum\limits_{k=1}^{+\infty} \left(\frac{1}{k} \ln^+ A_k\right)^{p+1} < +\infty$, $0 < p < +\infty$, then $\int_0^1 (1-r)^{p-1} \ln M_f(r) dr < +\infty$.*
2. *$\int_0^1 (1-r)^{p-1} \ln M_f(r) dr < +\infty$, $0 < p < +\infty \Longrightarrow \sum\limits_{k=1}^{+\infty} \left(\frac{1}{k} \ln^+ A_k\right)^{p+1} < +\infty$.*

Theorem 2. *Let $f \in \mathcal{A}^p_{+\infty}$ and $A_k = \max\{|a_n| : \|n\| = k\}$, $k \in \mathbb{Z}_+$.*

1. *If $\varkappa_{k+1} = A_k/A_{k+1} \nearrow +\infty$ $(k \uparrow +\infty)$, $0 < p < +\infty$ then*

$$\int_0^{+\infty} \frac{r^{p-1}}{\ln \ln M_f(r)} dr < +\infty \Longleftrightarrow \sum_{k=1}^{+\infty} \frac{1}{k \ln^2 k} \varkappa_k^p < +\infty.$$

2. *If $A_k/A_{k+1} \nearrow +\infty$ $(k \uparrow +\infty)$ and $\int_0^{+\infty} \dfrac{r^{p-1}}{\ln \ln M_f(r)} dr < +\infty$, $0 < p < +\infty$, then $\sum\limits_{k=1}^{+\infty} \dfrac{1}{k \ln^2 k} A_k^{-p/k} < +\infty$.*

Mean value theorems for polynomial solutions of linear elliptic equations with constant coefficients in the complex plane

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Let $B_R := \{z \in \mathbb{C} : |z| < R\}$, $m, n \in \mathbb{N}$, $s \in \mathbb{N}_0$, $n \geq 3$, $s < m < n+1$, $d_n := 2(5 + 4\cos\pi/n)^{-1/2}$ for odd n, and $d_n := 2(4 + 5\cos^2\pi/n)^{-1/2}$ for even n. Denote by $E(n, m, s)$ the set of all pairs of integer nonnegative numbers (k, l) such that the following conditions hold: $k < m - s$ or $l < m$; $k < n + s$; $l < n - s$.

Theorem 1. *Let $R > 0$, $f \in C^{2m-s-2}(B_R)$, $r \in (0, d_n R)$. Then the following assertions are equivalent:*

(*a*) *for all $z \in B_R$ and $\alpha \in [0,\ 2\pi)$ such that $\{z + re^{i\alpha + i2\pi\nu/n}\}_{\nu=0}^{n-1} \subset B_R$ we have the equality*

$$\sum_{p=s}^{m-1} \frac{r^{2p}}{(p-s)!p!} \partial^{p-s} \bar{\partial}^p f(z) = \sum_{\nu=0}^{n-1} (re^{i\alpha + i2\pi\nu/n})^s f(z + re^{i\alpha + i2\pi\nu/n});$$

(*b*) *the function f is represented in the form*

$$f(z) = \sum_{(k,l) \in E(n,m,s)} c_{k,l} z^k \bar{z}^l, \quad c_{k,l} \in \mathbb{C}. \tag{1}$$

It follows from the definition of the set $E(n, m, s)$ that functions satisfying the condition (1) form a finite-dimensional linear space over field $\mathbb{C}$, whose elements are polynomial solutions of the equation $\partial^{m-s}\overline{\partial}^m f = 0$.

Let $m = 1$ and $s = 0$. Then Theorem 1 is a well-known result proved by Remsey and Weit [1] in the case $R = \infty$ and by Volchkov [2, Part 5, Chapter 5, assertion (1) of Theorem 5.9] in the general case. These results contain the following classical theorem, which was proved independently in [3], [4], and [5, Chapter 3, §11]: a function $f \in C(\mathbb{C})$ is a harmonic polynomial of order $< n$ if and only if the mean value of the function f taken over vertices of any regular n-gon equals to the value of this function at its center.

1. T. Ramsey and Y. Weit, *Mean values and classes of harmonic functions*, Math. Proc. Camb. Phil. Soc. **96** (1984), 501–505.
2. V. V. Volchkov, *Integral Geometry and Convolution Equation*, Dordrecht–Boston–London: Kluwer Academic Publishers. 2003. 454 p.
3. S. Kakutani and M. Nagumo, *On the functional equation* $\sum_{\nu=0}^{n-1} f(z + e^{2\nu\pi i/n}\xi) = nf(z)$, Zenkoku Sûgaku Danwakai **66** (1935), 10–12 (in Japanese).
4. J. L. Walsh, *A mean value theorem for polynomials and harmonic polynomials*, Bull. Amer. Math. Soc. **42** (1936), 923–930.
5. I. I. Privalov, *Subharmonic functions*. Moscow–Leningrad, ONTI NKTP SSSR, 1937. 200 p.

The l-index boundedness of confluent hypergeometric limit function

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For a positive continuous function l on $[0, +\infty)$ an entire function f is said to be of bounded l-index, if there exists $N \in \mathbb{Z}_+$ such that for all $n \in \mathbb{Z}_+$ and $z \in \mathbb{C}$

$$\frac{|f^{(n)}(z)|}{n!l^n(|z|)} \leq \max\left\{\frac{|f^{(k)}(z)|}{k!l^k(|z|)} : 0 \leq k \leq N\right\}.$$

The least such integer N is called l-index and is denoted by $N(f, l)$.

The entire solution of the equation

$$zw'' + \alpha w' - w = 0, \quad \alpha \in \mathbb{C} \setminus \mathbb{Z}_-,$$

is said to be the confluent hypergeometric limit function and has power expansion

$$F(z) = F(\alpha; z) = 1 + \sum_{k=1}^{\infty} \left(\prod_{j=0}^{k-1} \frac{1}{j+\alpha}\right) \frac{z^k}{k!}.$$

Theorem. *If $\alpha > 0$ then for every $\eta \in (0, \ln 2)$ we have $N(F, l) \leq 1$ with*

$$l(|z|) \equiv \frac{e^{\eta}}{(2 - e^{\eta})\eta\alpha} + \frac{1 + \sqrt{4\ln 2 + 1}}{2\eta \min\{\alpha, 2\}}.$$

On exceptional set in Wiman's type inequality for entire functions of several variables

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By $\mathcal{E}^p$ we denote the class of entire functions $f(z) = \sum_{\|n\|=0}^{+\infty} a_n z^n$ such that for any $j \in \{1, 2, \ldots, p\}$: $\frac{\partial}{\partial z_j} f(z) \not\equiv 0$ in $\mathbb{C}^p$. Here $\|n\| = n_1 + \ldots + n_p$ $(p \geq 2)$, $z = (z_1, \ldots, z_p) \in \mathbb{C}^p$. For $r = (r_1, r_2, \ldots, r_p) \in \mathbb{R}^p_+$ and $f \in \mathcal{E}^p$ we denote

$$\Delta_r = \{t \in \mathbb{R}^p_+ \colon t_j \geq r_j, j \in \{1, \ldots, p\}\},$$

$$\mu_f(r) = \max\{|a_n| r^n \colon n \in \mathbb{Z}^p_+\},$$

$$M_f(r) = \max\{|f(z)| \colon |z_j| \leq r_j,\ j \in \{1, \ldots, p\}\}.$$

Let $h \in H^p$ i.e. be a positive non-deceasing in each variables function $h\colon \mathbb{R}^p_+ \to \mathbb{R}_+$ such that there exists $r_0 \in \mathbb{R}^p_+$, $\nu(\Delta_{r_0}) := \int_{\Delta r_0} h(r) \prod_{j=1}^p \frac{dr_j}{r_j} = +\infty$. We say that $E \subset \mathbb{R}^p$ is a *set of asymptotically finite h-logarithmic measure*, if there exists $r_0 \in \mathbb{R}^p$ such that

$$\nu_h(E \cap \Delta_{r_0}) := \int_{E\cap\Delta r_0} h(r) \prod_{j=1}^{p} \frac{dr_j}{r_j} < +\infty.$$

Theorem 1. *For every $f \in \mathcal{E}^p$ and any $\delta > 0$ there exists a set $E = E(f, \delta)$ of asymptotically finite h-logarithmic measure such that for all $r \notin E$*

$$M_f(r) \le \mu_f(r)(h(r))^{(p+1)/2+\delta} \ln^{p/2+\delta}(\mu_f(r)h(r)) \prod_{j=1}^{p} \ln^{p/2+\delta} r_j.$$

In the case $p = 1$ we find the following theorem.

Theorem 2 [1]**.** *Let $f \in \mathcal{E}^1$ be an entire function and $h \in H^1$ such that $\ln^+ \ln^+ h(r) = o(\ln\ln \mu_f(r))$ $(r \to +\infty)$. Then for every $\varepsilon > 0$ the inequality*

$$M_f(r) \le h(r)\mu_f(r)(\ln \mu_f(r))^{1/2+\varepsilon}$$

holds for every $r \in [1; +\infty) \setminus E(\varepsilon, f, h)$, where the set $E_3(\varepsilon, f, h) \equiv E_3$ is such that

$$h\text{-}meas\ E_3 := \int_{E_3} h(r) d\ln r < +\infty.$$

If $\ln h(r) = (\ln \mu_f(r))^a$, $a > 0$, then for any $\varepsilon > 0$ the inequality

$$M_f(r) \le h(r)\mu_f(r)(\ln \mu_f(r))^{1/2+1/2\max\{1,a\}+\varepsilon}$$

holds for every $r \in [1; +\infty) \setminus E^*(\varepsilon, f, h)$, where the set $E^*(\varepsilon, f, h) \equiv E^*$ is such that h-meas $E^* < +\infty$.

1. O. B. Skaskiv and O. V. Zrum, *On an exceptional set in the Wiman inequalities for entire functions*, Mat. Stud. **21**:1 (2004), 13–24.

Asymptotic behaviour of means of nonpositive $\mathcal{M}$-subharmonic functions

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We describe asymptotic behavior of pth means, $1<p<\frac{2n-1}{2(n-1)}$, of nonpositive $\mathcal{M}$-subharmonic functions in the unit ball B in $\mathbb{C}^n$ in terms of smoothness properties of the Riesz measure μ_u and its boundary measure.

Each nonpositive $\mathcal{M}$-subharmonic function $u \not\equiv -\infty$ on B has the following representation:

$$u(z) = -\int_S \left\{ \frac{1-|z|^2}{|1-\langle z,\xi\rangle|^2} \right\}^n d\nu(\xi) - \int_B G(z,w)d\mu_u(w),$$

where μ_u is the Riesz measure of u, ν is a nonnegative Borel measure on S and $G(z,w)$ is the invariant Green's function. Define

$$d\lambda(w) = \frac{4n^2}{n+1} d\nu(w) + (1-|w|^2)^n d\mu_u(w), \quad w \in \bar{B}.$$

For $\xi \in S$ and $\delta > 0$ we set $C(\xi,\delta) = \{z \in B : |1-\langle z,\xi\rangle| < \delta\}$.

Theorem. *Let u be a nonpositive $\mathcal{M}$-subharmonic function in B, $u \not\equiv -\infty$, $1<p<\frac{2n-1}{2(n-1)}$, $0 \le \gamma < 2n$. Then*

$$m_p(r,u) = O\left((1-r)^{\gamma-n}\right), \ r \uparrow 1$$

holds if and only if

$$\left(\int_S \lambda^p (C(\xi,\delta))\, d\sigma(\xi) \right)^{\frac{1}{p}} = O(\delta^\gamma), \ 0<\delta<1.$$

Complex geodesics in convex domains and $\mathbb{C}$-convexity of semitube domains

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The theorem presented in the talk provides certain conditions allowing to find formulas for all complex geodesics in a large family of convex domains in $\mathbb{C}^n$. This family contains, among others, all bounded convex domains and all convex semitube domains with no complex affine lines. The $\mathbb{C}$-convexity of semitube domains is also discussed.

Sandwich results for higher order derivatives of fractional derivative operator

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In this paper, we obtain some differential subordinations and superordinations related to a generalized fractional derivative operator for higher order derivatives of p-valent functions and also derive some sandwich results under certain assumptions on the parmmeters involved. These new results generalize some previously well-known results.

Constrained Gauss variational problems for vector measures associated with a condenser with intersecting plates

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We study a constrained Gauss variational problem relative to a positive definite kernel on a locally compact space for vector measures associated with a condenser $\mathbf{A} = (A_i)_{i\in I}$ whose oppositely charged plates intersect each other in a set of capacity zero. Sufficient conditions for the existence of minimizers are established, and their uniqueness and vague compactness are studied. Note that the classical (unconstrained) Gauss variational problem would be unsolvable in this formulation. We also analyze continuity of the minimizers in the vague and strong topologies when the condenser and the constraint both vary, describe the weighted equilibrium vector potentials, and single out their characteristic properties. Our approach is based on the simultaneous use of the vague topology and a suitable semimetric structure defined in terms of energy on a set of vector measures associated with $\mathbf{A}$, and on the establishment of completeness results for proper semimetric spaces. The theory developed is valid in particular for the classical kernels, which is important for applications. The study is illustrated by several examples.

Weighted generalizations of reproducing kernels

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In the beginning I will say something about reproducing kernels of weight-square-summable sequences Hilbert spaces. Then I will introduce the concept of weighted Szegő kernel. I will explain how the definition of an admissible weight in this case is different to the analogical one for Bergman kernel. Then I will give some information about how to calculate weighted Szegő kernel in specific situations. In the end, I will show how weighted Szegő kernel can be used to prove general theorems of complex analysis.

Satellite Symposium on Integrable Systems

Non-homogeneous hydrodynamic systems and quasi-Stackel Hamiltonians

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We propose a new algebraic concept of infinite hierarchies of monic polynomials, such that the coefficients of the polynomials in the next generation of the hierarchy are the zeros of a polynomial in the current generation. Using this concept, we formulate a new method to construct solvable N-body problems. By assuming that the zero generation – seed – polynomial has time-dependent coefficients that evolve according to a known solvable N-body system, we describe the motion of the coefficients of the polynomials in the subsequent generations of the polynomial hierarchy. This way an infinite hierarchy of N-body problems is constructed. Each system in the last hierarchy is solvable: its solution evolves as the N zeros of a polynomial with time-dependent coefficients that themselves evolve as the zeros of another polynomial, from a previous generation in the polynomial hierarchy, and so on, until the seed polynomial is reached. Among the examples, we present a new solvable N-body problem that is a hybrid of the two famous Calogero-Moser and goldfish models.

This is a joint work with Francesco Calogero.

Non-homogeneous hydrodynamic systems and quasi-Stackel Hamiltonians

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We present a novel construction of non-homogeneous hydrodynamic equations from what we call quasi-Stackel systems, that is non-commutatively integrable systems constructed from appropriate maximally superintegrable Stackel systems. We describe the relations between Poisson algebras generated by quasi-Stackel Hamiltonians and the corresponding Lie algebras of vector fields of non-homogeneous hydrodynamic systems. We also apply Stackel transform to obtain new nonhomogeneous equations of considered type.

On integrable discretizations of pseudospherical surfaces

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The discretization of pseudospherical surfaces in asymptotic coordinates is well known (Sauer, Wunderlich, Bobenko, Pinkall). It is natural to expect similar results in the case of curvature coordinates. Surprisingly enough, this case seems to be much more difficult, especially when the related problem of discretizing the sine-Gordon equation is concerned. Usually the construction of discrete curvature nets is based on discrete asymptotic nets. Our approach is different. We consider orthogonal nets derived directly from Lax pairs represented in a Clifford algebra.

Bonnet surfaces and matrix Lax pairs of Painleve' VI

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We build analytic surfaces in $\mathbb{R}^3$ represented by the most general sixth Painlevé equation P_{VI} in two steps. Firstly, the moving frame of the surfaces built by Bonnet in 1867 is extrapolated to a new, second order, isomonodromic matrix Lax pair of P_{VI}, whose elements depend rationally on the dependent variable and quadratically on the monodromy exponents θ_j. Secondly, by converting back this Lax pair to a moving frame, we obtain an extrapolation of Bonnet surfaces to surfaces with two more degrees of freedom.

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Integrable discrete systems with sources

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We show that the discrete Kadomtsev-Petviashvili equation with sources, recently obtained by the "source generalization" method, can be incorporated into the squared eigenfunctions symmetry extension procedure. Moreover, using the known correspondence between Darboux-type transformations and additional independent variables we uncover the origin of the source terms as coming from multidimensional consistency of the Hirota system itself. This helps us to construct Darboux-type transformations (elementary, adjoint and the fundamental one) for the system with sources. Finally, we briefly present simiar results for the discrete Darboux (N-wave) system and its C-symmetric reduction.

Quantization on the cotangent bundle of a Lie group

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We demonstrate a quantization procedure of a classical Hamiltonian system defined on the cotangent bundle of a Lie group. We use the formalism of deformation quantization, which describes quantization as an appropriate deformation of the classical system. We also construct an operator representation of the received quantum system. The developed theory is illustrated on the example of quantizing a rigid body. In particular we consider quantum versions of such Liouville integrable systems as Euler's top. We also investigate phase space reductions of the considered quantum systems focusing on the reduction of the phase space to the dual of a Lie algebra.

The Lax-Sato integrable heavenly equations on functional manifolds and supermanifolds and their Lie-algebraic structure

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In the paper [1] the general Lie-algebraic approach to constructing the Lax-Sato integrable dispersionless heavenly systems on functional manifolds has been developed. It is based on the Adler-Kostant-Symes theory and $\mathcal{R}$-operator structures related with the loop Lie algebra $\widetilde{diff}(\mathbb{T}^n)$ of vector fields on the n-dimensional torus $\mathbb{T}^n$ and adjacent holomorphic in the "spectral" parameter $\lambda \in \mathbb{S}^1_{\pm} \subset \mathbb{C}$ Lie algebra $diff_{hol}(\mathbb{C} \times \mathbb{T}^n) \subset diff(\mathbb{C} \times \mathbb{T}^n)$ of vector fields on $\mathbb{C} \times \mathbb{T}^n$.

In the framework of this approach the Lax-Sato integrable heavenly systems arise as a compatible condition for two commuting Hamiltonian flows on the dual spaces to the Lie algebras $\widetilde{diff}(\mathbb{T}^n)$ and $diff_{hol}(\mathbb{C} \times \mathbb{T}^n)$ and their conservation laws are generated by the corresponding sets of Casimir invariants.

In our report the Lie-algebraic description of such known systems as first and second reduced Shabat type [2] and Hirota [3] heavenly equations by use of the loop Lie algebra $\widetilde{diff}(\mathbb{T}^1)$ is proposed. The existence of a connection between the Liouville type [4] equations and loop Lie algebra $\widetilde{diff}(\mathbb{T}^1_{\mathbb{C}})$ of the vector fields on the complex torus $\mathbb{T}^1_{\mathbb{C}}$ is established by means of some linear mapping of the "spectral" parameter $z \in \mathbb{T}^1_{\mathbb{C}}$.

In the case of $n = 1$ a generalization of the Lie-algebraic scheme of [1], related with the loop Lie algebra $\widetilde{diff}(\mathbb{S}^{1|N})$ of superconformal vector fields on the $1|N$-dimensional supertorus $\mathbb{S}^{1|N} \simeq \mathbb{S}^1 \times \Lambda_1^N$, where $\Lambda := \Lambda_0 \oplus \Lambda_1$ is an infinite-dimensional Grassmann algebra over $\mathbb{C}$, $\Lambda_0 \supset \mathbb{C}$, is considered. This generalized scheme is applied to construct the Lax-Sato integrable superanalogs of the Mikhalev-Pavlov [5, 6] heavenly equation for every $N \in \mathbb{N} \setminus \{4; 5\}$. By use the loop Lie algebra $\widetilde{diff}(\mathbb{S}^{1|N})$ the integrable superanalogs of the Liouville type equations are obtained as a result of some superconformal mapping in the space of variables $(z, \theta_1, \ldots, \theta_N) \in \mathbb{S}^{1|N}$.

1. O. E. Hentosh, Ya. A. Prykarpatsky, D. Blackmore, and A. K. Prykarpatski, *Lie-algebraic structure of Lax-Sato integrable heavenly equations and the Lagrange-d'Alembert principle*, J. Geometry and Physics **120** (2017), 208–227.
2. L. M. Alonso and A.B. Shabat, *Hydrodynamic reductions and solutions of an universal hierarchy*, Theor. Math. Phys, **104** (2004), 1073–1085.
3. M. Dunajski and W. Kryǹski, *Einstein-Weyl geometry, dispersionless Hirota equation and Veronese webs*, arXiv:1301.0621v2 [math.DG] 17 Jan 2014, 11 p.
4. L. V. Bogdanov and B.G. Konopelchenko, *Grassmannians $Gr(N-1, N+1)$, closed differential $N-1$ forms and N-dimensional integrable systems*, arXiv:1208.6129v2 [nlin.SI] 5 Mar 2013, 26 p.
5. V. G. Mikhalev, *On the Hamiltonian formalism of Korteweg-de Vries type hierarchies*, Funct. Anal. Appl. **26**:2 (1992), 140–142.
6. M. V. Pavlov, *Integrable hydrodynamic chains*, J. Math. Phys. **44**:9 (2003), 4134–4156.

The eigenvalues of Laplace operator with Dirichlet-type boundary conditions for sphere-conical resonator connected with continuum through the circular hole

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The problem corresponds to the electromagnetic excitation of the perfectly conducting sphere-conical resonator which is connected to the free space through the circular hole. The problem is formulated in terms of scalar potential for spherical coordinate system as a mixed boundary problem for Helmholtz equation. The unknown potential is sought as expansion in series of eigenfunctions for each region, formed by the sphere-conical cavity. Then the solution to the problem is reduced to the infinite set of linear algebraic equations by means of mode matching technique and orthogonality properties of the eigen functions. The main part of asymptotic of matrix elements determined for large indexes identifies the convolution type operator. The corresponding inverse operator is represented in the explicit form. Both these operators are applied to reduce the problem to the linear algebraic equations of the second kind. The explicit solution to this system is obtained for the small hole of the cavity. Based on this the complex value of perturbation of the real resonant frequency of the closed sphere-conical volume caused by the small hole is determined. The radiation characteristics for the cavity with arbitrary radius of the circular hole excited by the resonant frequencies are examined numerically.

Bi-homogeneity and integrability of rational potentials

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We consider natural Hamiltonian systems with two degrees of freedom for which Hamiltonian function is $H = \frac{1}{2}(p_1^2+p_2^2)+V(q_1, q_2)$ and potential $V(q_1, q_2)$ is a rational function. Necessary conditions for the integrability of such systems are deduced from integrability of the dominant term of the potential which usually is an appropriately chosen homogeneous term of V. We show that introducing weights compatible with the canonical structure we can find new dominant terms which can give new necessary conditions for integrability. To deduce them we investigate integrability of 2-integer parameters family of bi-homogeneous potentials. Unexpectedly systems with these potentials can be reduced to the Lotka-Volterra quadratic planar vector field. Then, theorem of Jean Moulin Ollagnier, allows us to make complete classification of integrable cases. Among these integrable systems one is integrable with additional first integral of degree four with respect to the momenta. We explicitly integrate this system. Its solution are algebraic functions of the Weierstrass elliptic function and linear function of time.

Differential geometry and well-posedness of the KP hierarchy

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We describe how the notion of diffeology can apply to Mulase constructions in the KP hierarchy. The existence and the uniqueness of the solution is here re-interpreted with rigorous differential geometric constructions, by two different approaches. The first one is based on the algebraic approach of Reyman and Semenov-tian-Shansky, and the second one describes the solutions of the initial KP hierarchy as the infinite jet of a global section of a principal bundle. These two approaches show very naturally that the solutions depend smoothly of the initial conditions and time. With these

precisions on the adequate setting, we will finish with the last developments and open questions of this research program still in progress .

Deformations of functions on surfaces by area preserving diffeomorphisms

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Let M be a compact orientable surface equipped with a volume form ω, P be either $\mathbb{R}$ or S^1, $f : M \to P$ be a C^∞ Morse map, and H be the Hamiltonian vector field of f with respect to ω. Let also $\mathcal{Z}_\omega(f) \subset C^\infty(M, \mathbb{R})$ be set of all functions taking constant values along orbits of H, and $\mathcal{S}_{\mathrm{id}}(f, \omega)$ be the identity path component of the group of diffeomorphisms of M mutually preserving ω and f.

We construct a canonical map $\varphi : \mathcal{Z}_\omega(f) \to \mathcal{S}_{\mathrm{id}}(f, \omega)$ being a homeomorphism whenever f has at least one saddle point, and an infinite cyclic covering otherwise. In particular, we obtain that $\mathcal{S}_{\mathrm{id}}(f, \omega)$ is either contractible or homotopy equivalent to the circle.

Similar results hold in fact for a larger class of maps $M \to P$ whose singularities are equivalent to homogeneous polynomials without multiple factors.

Paper reference: (**http://arxiv.org/abs/1701.03509**).

On the constant astigmatism equation

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We shall discuss the constant asigmatism equation, its nonlocal transformations, known solutions, and the structure of nonlocal conservation laws.

Paper reference: J. Geom. Phys. **113** (2017), 117–130.

Local bisymplectic realizations of compatible Poisson brackets

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In a seminal paper "The local structure of Poisson manifold" (1983) A. Weinstein proved that for any Poisson manifold (M, P) there exists a local symplectic realization, i.e. nondegenerate Poisson manifold (M', P') and a local surjective submersion $f : M' \to M$ with $f_* P' = P$. Global aspects of this problem were afterwards intensively studied as they are related to the theory of symplectic and Poisson grupoids, to the integration problem of Lie algebroids, and to different quantization schemes.

In this talk I will discuss a problem of local simultaneous realization of two compatible Poisson structures by means of two nondegenerate ones. Note the following essential difference between the two realization problems: there is only one local model of the nondegenerate Poisson bivector P' given by the Darboux theorem and there are many local models of bisymplectic bihamiltonian structures. Thus, besides the problem of existence it is important to understand how many nonequivalent realizations there are in the second case.

Paper reference: (**http://arxiv.org/abs/1705.03522**).

Lax representations for matrix short pulse equations

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The Lax representation for different matrix generalizations of Short Pulse Equations (SPE) is considered. The four-dimensional Lax representations of four-component Matsuno, Feng and Dimakis-Müller-Hoissen-Matsuno equations is obtained. The four-component Feng system is defined by generalization of the two-dimensional Lax representation to the four-component case. This system reduces to the original Feng equation or to the two-component Matsuno equation or to the Yao-Zang equation. The three component version of Feng equation is presented. The four-component version of Matsuno equation with its Lax representation is given . This equation reduces the new two-component Feng system. The two-component Dimakis-Müller-Hoissen-Matsuno equations are generalized to the four parameter family of the four-component SPE. The bi-Hamiltonian structure of this generalization, for special values of parameters, is defined. This four-component SPE in special case reduces to the new two-component SP.

Paper reference: Paper reference: (**http://arxiv.org/abs/1705.04030**).

The classical M. A. Buhl problem, its Pfeiffer–Sato solutions and the Lagrange–d'Alembert principle for integrable heavenly nonlinear equations

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The work is devoted to old and recent investigations of the classical M.A. Buhl problem of describing compatible linear vector field equations and its general M.G. Pfeiffer and Lax–Sato type special solutions. In particular, we analyze the related Lie-algebraic structures and integrability properties of an interesting class of nonlinear dynamical systems called the dispersionless heavenly equations, which were initiated by Plebański and later analyzed in a series of articles. The AKS-algebraic and related $\mathcal{R}$-structure schemes are used to study the orbits of the corresponding coadjoint actions, which are intimately related to their classical Lie–Poisson structures. It is demonstrated that their compatibility condition coincides with the corresponding heavenly equations under consideration. Moreover, it is shown that all these equations arise in this way and can be represented as a Lax compatibility condition for specially constructed loop vector fields on the torus. The infinite hierarchy of conservation laws related to the heavenly equations is described, and its analytical structure connection with the Casimir invariants is mentioned. In addition, typical examples of such equations, demonstrating their integrability via the scheme devised herein, are presented. The relationship of an interesting Lagrange–d'Alembert type mechanical interpretation of the devised integrability scheme with the Lax–Sato equations is also discussed.

Dynamics of rigid bodies with movable points

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Recently a lot of work has been devoted to study N materials points in spaces of constant positive curvature S^2 and S^3 interacting according to analogues of Newtonian law of gravitation or Hooke's law of elasticity. Whereas less is known about dynamics of material points on sphere whose motion is subjected to certain holonomic constraints e.g. fixed arc lengths between certain number of material points. We will show that dynamics of a few material points on sphere S^2 with a certain number of such constraints is equivalent to dynamics of a rigid body with a fixed point with some additional internal degrees of freedom. In particular dynamics of two material points on sphere with fixed arc length between them is equivalent to the dynamics of a rigid body with one fixed point and specific moments of inertia. A system of three

points on a sphere constrained such that the distances between two their pairs are fixed is equivalent to a rigid body with one additional point which can move freely along a circle fixed in the body. This last system can be generalized to a rigid body with a fixed point containing a several points which can move along prescribed curves inside this rigid body. Equations of motion of such system after reduction by their symmetry are Hamiltonian with a certain degenerated Poisson structure. In particular a rigid body with point masses moving along a straight line and on a circle will be considered. Some integrable cases are identified and non-integrability proofs by means of differential Galois theory are given.

Rattleback versus tippe top. How do they move and why?

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The rattleback and the tippe top are two toy rigid bodies displaying counterintuitive behaviour when they move on a flat surface. The rattleback spinned in the "wrong" direction starts to rattle, slows down and acquires rotation in the opposite, preferred sense of spinning. The tippe top, when it is spun fast in any sence of direction starts to nutate and inverts to continue spinning on the handle untill it falls down due to frictional loss of energy.

It appears that the sliding friction is necessary for the inversion of the tippe top while reversion of the rattleback can well be explained with the use of pure rolling model equations that conserve energy.

I shall explain how we may understand motion of both bodies using the underlying mechanical equations for rolling and sliding rigid bodies and how to get insight into their dynamics using numerical simulations.

New Bihamiltonian Generalizations of NSE

Yuriy Sydorenko

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We consider a generalization of some non-linear two-component bi-Hamiltonian evolution systems of differential equations for vector (multicomponent) generalizations case with saving Lax integrability. Also we suggest that the new generalization of non-linear Kaup-Newell models allows the integro-differential Lax representation.In addition, we propose the method of multicomponent generalizations of integrable systems based on their bi-Hamiltonicity.

New obtained integrable systems include vector generalizations of well-known Chen-Lee-Liu, Newell-Sigur, Kundu-Eckhaus non-linear models and Gerdjikov-Ivanov equations. The mentioned models are named as "the second type generalization" to distinguish them from previously known models of other various reseachers, which we call "the first type generalization".

Hierarchies of Manakov-Santini type

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I will talk about construction of ManakovвЂ“Santini type hierarchies of hydrodynamic integrable equations. The formalism is based on the LaxвЂ“Sato approach, where integrability conditions are the RotaвЂ“Baxter identity and some other identity that was introduced by me in the context of construction of Frobenius manifolds within classical r-matrix formalism.

I will illustrate the theory by means of the algebra of Laurent series and the related (2+1)-dimensional equations of ManakovвЂ“Santini type. I will also discuss reduction on the more standard hierarchies of dispersionless systems.

The talk will mainly be based on the article published in SIGMA **12** (2016) 022.

Integrability of natural Hamiltonian systems in curved spaces

Wojciech Szumiński

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We investigate a class of natural Hamiltonian systems with two degrees of freedom with the general form of a metric. Using a particular solution we derive necessary conditions for the integrability of this system investigating differential Galois group of variational equations.

Some properties of Kahler-Norden-Codazzi golden structures on pseudo-Riemannian manifolds

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In this study we consider Kahler-Norden-Codazzi Golden structures on pseudo-Riemannian manifolds and study curvature properties of them. Also we define special connections on the manifold and investigate some its properties.

On discretization of Darboux integrable equations

Kostyantyn Zheltukhin and Natalya Zheltukhina

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We consider the discretization, preserving integrability, of Darboux integrable hyperbolic type equations. In particular, we study the discretization of Laine equations. For integrals of each Laine equation, we find a semi-discrete equation, admitting that integral as an n-integral. We also prove that constructed semi-discrete equations turn out to be Darboux integrable.

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International Conference in Functional Analysis dedicated to the 125th anniversary of Stefan Banach

Lviv, Ukraine, 18-23 September 2017

Book of Abstracts

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