The 14th Summer School

ANALYSIS, TOPOLOGY, ALGEBRA and APPLICATIONS

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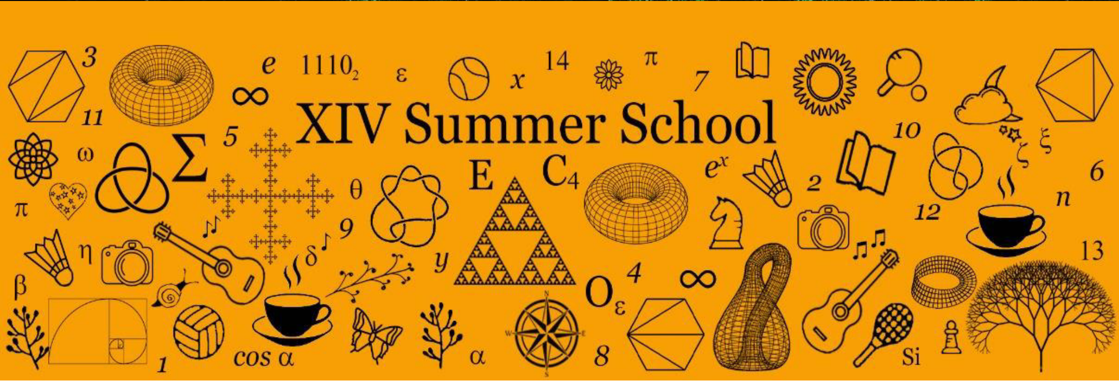
Pidzakharychi, Chernivtsi Region, Ukraine

BOOK OF ABSTRACTS



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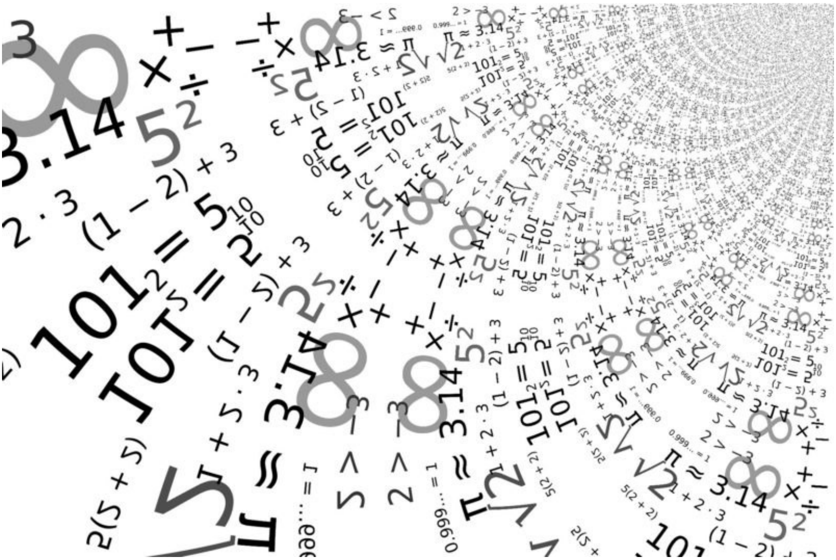

Duality between coarse and uniform spaces via function algebras and group actions

Taras Banakh

Jan Kochanowski University in Kielce, Poland

and

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We establish the functorial duality between four categories: of coarse spaces, of group acts, of topologically discrete totally bounded uniform spaces, and of compact Hausdorff spaces with dense set of isolated points. Using these dualities we shall prove that any compact Hausdorff space X with sequential square and dense set X' of isolated point is equal to the Higson compactification of X' endowed with a finitary coarse structure generated by the group $Homeo(X, X')$ of homeomorphisms of X that do not move points of the set $X \setminus X'$. Consequently, each compact metrizable space is homeomorphic to the Higson corona of some finitary coarse space.

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CARDINAL CHARACTERISTICS OF THE LATTICE OF SHIFT-CONTINUOUS TOPOLOGIES ON THE BICYCLIC MONOID WITH AN ADJOINED ZERO

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We investigate the upper semilattice $\mathcal{IST}_{lc}$ of Hausdorff inverse semigroup topologies on the bicyclic semigroup with an adjoined zero which induce a locally compact topology on $E(\mathcal{C}^0)$. We give a powerful method of constructing inverse semigroups topologies on $\mathcal{C}^0$ using subfamilies of $\omega^{\mathbb{Z}}$ which satisfy certain conditions. It was proved that there exists the coarsest inverse semigroup topology and the finest non-discrete inverse semigroup topology on $\mathcal{C}^0$. We show that $\mathcal{IST}_{lc}$ contains a well-ordered chain of cardinality $\mathfrak{b}$ and an antichain of cardinality $\mathfrak{c}$.

A Hausdorff topology τ on the bicyclic monoid $\mathcal{C}^0$ is called weak if it is contained in the coarsest inverse semigroup topology on $\mathcal{C}^0$. We introduce a notion of a shift-stable filter on ω and show that the lattice $\mathcal{W}$ of all weak shift-continuous topologies on $\mathcal{C}^0$ is isomorphic to the lattice $\mathcal{SS}^1 \times \mathcal{SS}^1$ where $\mathcal{SS}^1$ is the set of all shift-stable filters on ω with an attached element 1 endowed with the following partial order: $\mathcal{F} \leq \mathcal{G}$ iff $\mathcal{G} = 1$ or $\mathcal{F} \subset \mathcal{G}$. Also, we investigate cardinal characteristics of the lattice $\mathcal{W}$. In particular, we proved that the lattice $\mathcal{W}$ contains an antichain of cardinality $2^{\mathfrak{c}}$ and a well-ordered chain of cardinality $\mathfrak{t}$.

The work of the author was supported by the Austrian Science Fund FWF (Grant I 3709 N35).

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Communication complexity for mathematicians

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Communication complexity is one of the most interesting – that is, one of the strongest – computational models where meaningful mathematical analysis currently exists.

The aim of our presentation will be to start from the definitions, go through several known examples of exponential advantage of quantum over classical communication, and touch upon a few of the most important open questions in the area.

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On superextensions of semigroups and their automorphism groups

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A family $\mathcal{M}$ of non-empty subsets of a set X is called *an upfamily* if for each set $A \in \mathcal{M}$ any subset $B \supset A$ of X belongs to $\mathcal{M}$. By $\upsilon(X)$ we denote the set of all upfamilies on a set X. Each family $\mathcal{B}$ of non-empty subsets of X generates the upfamily $\langle\mathcal{B}\rangle = \{A \subset X : \exists B \in \mathcal{B}\ (B \subset A)\}$. An upfamily $\mathcal{F}$ that is closed under taking finite intersections is called a *filter*. A filter $\mathcal{U}$ is called an *ultrafilter* if $\mathcal{U} = \mathcal{F}$ for any filter $\mathcal{F}$ containing $\mathcal{U}$. The family $\beta(X)$ of all ultrafilters on a set X is called the *Stone-Čech compactification* of X, see [14]. An ultrafilter $\langle\{x\}\rangle$, generated by a singleton $\{x\}$, $x \in X$, is called *principal*. Each point $x \in X$ is identified with the principal ultrafilter $\langle\{x\}\rangle$ generated by the singleton $\{x\}$, and hence we can consider $X \subset \beta(X) \subset \upsilon(X)$. It was shown in [8] that any associative binary operation $* : S \times S \to S$ can be extended to an associative binary operation $* : \upsilon(S) \times \upsilon(S) \to \upsilon(S)$ by the formula

$$\mathcal{L} * \mathcal{M} = \Big\langle \bigcup_{a \in L} a * M_a : L \in \mathcal{L},\ \ \{M_a\}_{a \in L} \subset \mathcal{M} \Big\rangle$$

for upfamilies $\mathcal{L}, \mathcal{M} \in \upsilon(S)$. In this case the Stone-Čech compactification $\beta(S)$ is a subsemigroup of the semigroup $\upsilon(S)$. The semigroup $\upsilon(S)$ contains as subsemigroups many other important extensions of S. In particular, it contains the semigroup $\lambda(S)$ of maximal linked upfamilies. An upfamily $\mathcal{L}$ of subsets of S is said to be *linked* if $A \cap B \neq \emptyset$ for all $A, B \in \mathcal{L}$. A linked upfamily $\mathcal{M}$ of subsets of S is *maximal linked* if $\mathcal{M}$ coincides with each linked upfamily $\mathcal{L}$ on S that contains $\mathcal{M}$. It follows that $\beta(S)$ is a subsemigroup of $\lambda(S)$. The space $\lambda(S)$ is well-known in General and Categorial Topology as the *superextension* of S, see [16].

Given a semigroup S we shall discuss the algebraic structure of the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of S. We show that any automorphism of a semigroup S can be extended to an automorphism of its superextension $\lambda(S)$, and the automorphism group $\mathrm{Aut}\,(\lambda(S))$ of the superextension $\lambda(S)$ of a semigroup S contains a subgroup, isomorphic to the group $\mathrm{Aut}\,(S)$.

Proposition 1. *For any group G, each automorphism of $\lambda(G)$ is an extension of an automorphism of G.*

Theorem 1. *Two groups are isomorphic if and only if their superextensions are isomorphic.*

A semigroup S is called *monogenic* if it is generated by some element $a \in S$ in the sense that $S = \{a^n\}_{n \in \mathbb{N}}$. If a monogenic semigroup is infinite, then it is isomorphic to the additive semigroup $\mathbb{N}$ of positive integer numbers. A finite monogenic semigroup $S = \langle a \rangle$ also has simple structure. There are positive integer numbers r and m called the *index* and the *period* of S such that

- $S = \{a, a^2, \dots, a^{r+m-1}\}$ and $r + m - 1 = |S|$;
- $a^{r+m} = a^r$;
- $C_m := \{a^r, a^{r+1}, \dots, a^{r+m-1}\}$ is a cyclic and maximal subgroup of S with the neutral element $e = a^n \in C_m$ and generator a^{n+1}, where $n \in (m \cdot \mathbb{N}) \cap \{r, \dots, r+m-1\}$.

By $\mathrm{M}_{r,m}$ we denote a finite monogenic semigroup of index r and period m.

Theorem 2. *Two finite monogenic semigroups are isomorphic if and only if their superextensions are isomorphic.*

Proposition 2. *If $r \geq 3$, then any automorphism ψ of the semigroup $\lambda(\mathrm{M}_{r,m})$ has $\psi(x) = x$ for all $x \in \mathrm{M}_{r,m}$.*

For the idempotent e of the maximal subgroup C_m of a semigroup $\mathrm{M}_{r,m}$ the shift $\rho : \mathrm{M}_{r,m} \to e\mathrm{M}_{r,m} = C_m$, $\rho : x \mapsto ex$, is a homomorphic retraction of $\mathrm{M}_{r,m}$ onto C_m. Therefore, $\bar{\rho} = \lambda\rho : \lambda(\mathrm{M}_{r,m}) \to \lambda(C_m) \subset \lambda(\mathrm{M}_{r,m})$ is a homomorphic retraction as well.

Theorem 3. *For $r = 2$ the homomorphic retraction $\bar{\rho} : \lambda(\mathrm{M}_{r,m}) \to \lambda(C_m)$ has the following properties:*

1. $\mathcal{A} * \mathcal{B} = \bar{\rho}(\mathcal{A}) * \mathcal{B} = \mathcal{A} * \bar{\rho}(\mathcal{B}) = \bar{\rho}(\mathcal{A}) * \bar{\rho}(\mathcal{B})$ *for any* $\mathcal{A}, \mathcal{B} \in \lambda(\mathrm{M}_{r,m})$;
2. $\psi(x) = x$ *for any* $x \in C_m$ *and any* $\psi \in \mathrm{Aut}\,(\lambda(\mathrm{M}_{r,m}))$;
3. *the restriction operator* $R : \mathrm{Aut}\,(\lambda(\mathrm{M}_{r,m})) \to \mathrm{Aut}\,(\lambda(C_m))$ *has kernel isomorphic to* $\prod_{\mathcal{L}\in\lambda(C_m)} S_{\bar{\rho}^{-1}(\mathcal{L})\setminus\{\mathcal{L}\}}$ *and the range*

$$R(\mathrm{Aut}\,(\mathrm{M}_{r,m})) = \{\varphi \in \mathrm{Aut}\,(\lambda(C_m)) : \forall \mathcal{L} \in \lambda(C_m)\ \ |\bar{\rho}^{-1}(\varphi(\mathcal{L}))| = \\ = |\bar{\rho}^{-1}(\mathcal{L})|\}.$$

Consider the shift $\sigma : \mathrm{M}_{r,m} \to a\mathrm{M}_{r,m}$, $\sigma : x \mapsto ax$.

Theorem 4. *Assume that* $r \geq 2$. *The restriction operator* $R : \mathrm{Aut}\,(\lambda(\mathrm{M}_{r,m})) \to \mathrm{Aut}\,(\lambda(\mathrm{M}^{\cdot 2}_{r,m}))$ *has kernel isomorphic to*

$$\prod_{\mathcal{L}\in\lambda(\mathrm{M}^{\cdot 2}_{r,m})} S_{\bar{\sigma}^{-1}(\mathcal{L})\setminus\lambda(\mathrm{M}^{\cdot 2}_{r,m})}$$

and range $R(\mathrm{Aut}\,(\mathrm{M}_{r,m})) \subset H$ *where*

$$H = \big\{\varphi \in \mathrm{Aut}\,(\lambda(\mathrm{M}^{\cdot 2}_{r,m})) :\ \forall \mathcal{L} \in \lambda(\mathrm{M}^{\cdot 2}_{r,m}) \varphi(\bar{\sigma}^{-1}(\mathcal{L}) \cap \lambda(\mathrm{M}^{\cdot 2}_{r,m})) = \\ = \bar{\sigma}^{-1}(\mathcal{L}) \cap \lambda(\mathrm{M}^{\cdot 2}_{r,m}) \text{ and } \forall C \in \Xi_{\lambda(\mathrm{M}_{r,m})}\ |\bar{\sigma}^{-1}(\varphi(\mathcal{L})) \cap C| = \\ = |\bar{\sigma}^{-1}(\mathcal{L}) \cap C|\big\}.$$

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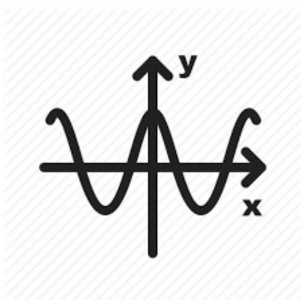

The algebraic face of the Collatz conjecture

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The $3n+1$ problem, also known as *the Collatz conjecture, the Syracuse problem*, is a conjecture in number theory stablished in 1937 by Lothar Collatz and can be stated as follows: If $f : \mathbb{N} \to \mathbb{N}$ is the function define by:

$$f(n) = \begin{cases} n/2, & n \text{ is even} \\ 3n+1, & n \text{ is odd,} \end{cases}$$

then given $n \in \mathbb{N}$, there exists $k > 0$ such that $f^{(k)}(n) = 1$ and the only orbit is $\{1, 2, 4\}$. In the sequel, this function will be called the Collatz function.

In this work we give an algebraic approach to this problem. In fact we prove that the conjecture is true if and only if the semiring τ_f has a unique maximal ideal, τ_f being the topology on $\mathbb{N}$ given by the open sets as those subset θ of $\mathbb{N}$ such that $f^{-1}(\theta) \subset \theta$, where f is the Collatz function.

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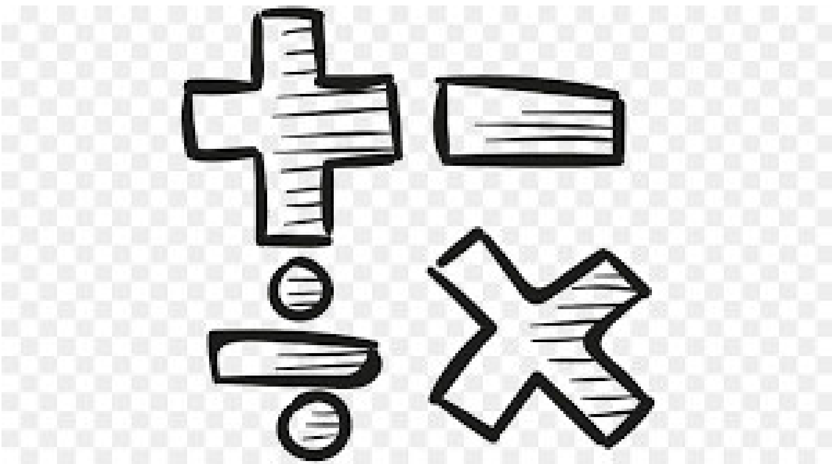

An algerba of uniformly continuous analytic functions on balls of fixed radius

Anna Hihliuk and **Andriy Zagorodnyuk**
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Let X be a separable complex Banach space and G be a countable dense subset of X. Let $P(X)$ denote the space of all continuous homogeneous polynomials endowed with the norm

$$\|P\|_z = \sup_{\|x-z\|\leq 1} |P(x)|$$

for all $z \in G$.

The metric generated by the countable system of norms is given by

$$\rho(x,y) = \sum_{n=1}^{\infty} \frac{\|x-y\|_z}{2^n(1+\|x-y\|_z)}. \tag{1}$$

We will consider a completion of the space $P(X)$ and denote it by $H_{UG}(X)$. Then $H_{UG}(X)$ becomes a Frechet algebra.

Theorem 1. *Let X be a separable complex Banach space and G be a countable dense subset of X. Let the alebra $H_{UG}(X)$ be a completion of the space $P(X)$ endowed with metric* (1)*. Then a function $f \in H_{UG}(X)$ if and only if f is a analytic function, which is uniformly continuous on every unit ball centered at $z \in G$.*

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Students' scientific activity at mathematical seminars and at the Student Scientific Society at the Lwów University (1894 – 1939)

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The mathematical seminar, which was organized in 1893 by Józef Puzyna, and the student physical-mathematical society began to form traditions of Lwów Mathematical School.

In our talk we will discuss the themes of works written by the members of the seminar and the student physical-mathematical society. The biographies of members of the seminars will be described as well.

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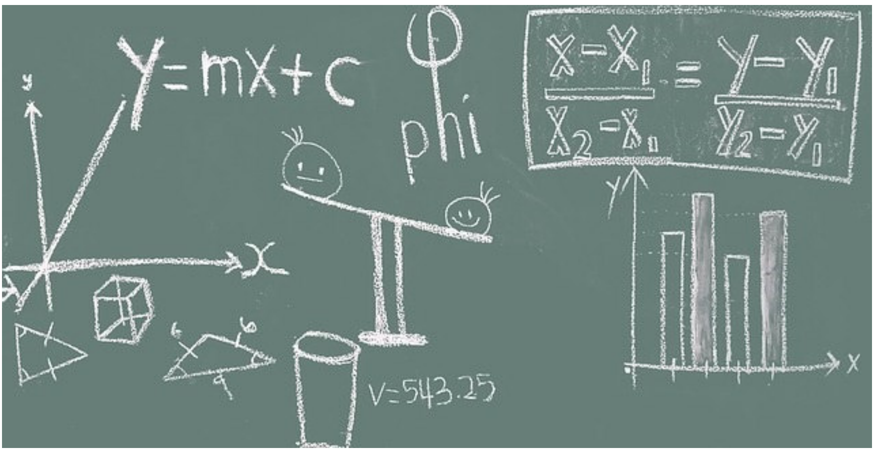

Strongly separately continuous functions and Borel subsets of ℓ_p

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and Yurij Fedkovych Chernivtsi National University, Ukraine

A notion of strongly separately continuous function of n real variables was introduced by Dzagnidze in [2] and studied in many papers (see [1, 3] and the literature given there).

Let $X_T = \prod_{t\in T} X_t$ be a product of family of sets X_t with $|X_t| > 1$ for all $t \in T$. If $S \subseteq S_1 \subseteq T$, $a = (a_t)_{t\in T}$ is a point of X_T and $x = (x_t)_{t\in S_1} \in \prod_{t\in S_1} X_t$, then x_S^a means a point $(y_t)_{t\in T}$ such that

$$y_t = \begin{cases} x_t, & t \in S, \\ a_t, & t \in T \setminus S. \end{cases}$$

In the case $S = \{s\}$ we will write x_s^a instead of $x_{\{s\}}^a$.

We say that a subset $A \subseteq X_T$ is *$\mathscr{S}$-open* if

$$\{y = (y_t)_{t\in T} \in X_T : |\{t \in T : y_t \neq x_t\}| \leq 1\} \subseteq A$$

for all $x = (x_t)_{t\in T} \in A$.

Let $X \subseteq X_T$ be an $\mathscr{S}$-open set, $\mathscr{T}$ be a topology on X and (Y, d) be a metric space. A function $f : (X, \mathscr{T}) \to Y$ is called *strongly separately continuous on* X or *an ssc-function* if

$$\lim_{x\to a} d(f(x), f(a_t^x)) = 0$$

for every $a \in X$ and $t \in T$.

In our presentation we will study the discontinuity points set of ssc-functions and connection of ssc-functions with the box-topology of infinite products. Moreover, the problem of Baire classification of strongly separately continuous functions will lead us to a construction of an $\mathscr{S}$-open subset of ℓ_p which belongs to the α'th additive Borel class and does not belong to the α'th multiplicative Borel class.

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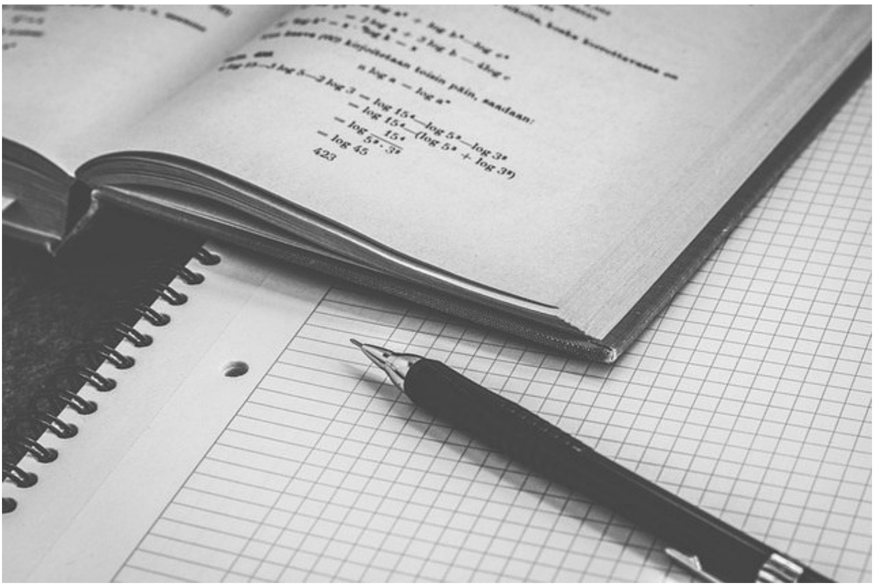

A Riemann-Hilbert problem approach to the modified Camassa-Holm equation on a nonzero background

Iryna Karpenko and **Dmitry Shepelsky**

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The modified Camassa–Holm equation has the following form:

$$m_t + \left((u^2 - u_x^2)m\right)_x = 0, \quad m = u - u_{xx}. \tag{1}$$

In an equivalent form, this equation was given by Fokas in [1] (see also [4] and [3]) and has attracted considerable interest since it was rederived by Qiao [2]. So it is sometimes referred to as the Fokas-Olver-Rosenau-Qiao equation. Equation (1) has a bi-Hamiltonian structure [4] and possesses a Lax pair [2].

We consider the initial value problem for the mCH equation (1):

$$m_t + \left((u^2 - u_x^2)m\right)_x = 0, \quad m = u - u_{xx}, \;\; t > 0, -\infty < x < +\infty, \tag{2}$$

$$u(x,0) = u_0(x), \qquad -\infty < x < +\infty, \tag{3}$$

assuming that $u_0(x) \to 1$ as $|x| \to \infty$, and we search for the solution that preserves this behavior: $u(x,t) \to 1$ as $|x| \to \infty$ for all $t > 0$.

We present the inverse scattering transform (IST) approach for the mCH equation using the formalism of 2×2 matrix Riemann–Hilbert problems formulated in the complex plane of the spectral parameter (cf. [5]). This approach is applied to the Lax pair of the mCH equation:

$$\begin{cases} \Phi_x(x,t,\lambda) = U(x,t,\lambda)\Phi(x,t,\lambda) \\ \Phi_t(x,t,\lambda) = V(x,t,\lambda)\Phi(x,t,\lambda) \end{cases},$$

where

$$A = \lambda^{-2} + \frac{(u^2 - u_x^2 + 2u)}{2},$$

$$B = -\lambda^{-1}(u - u_x + 1) - \frac{\lambda(u^2 - u_x^2 + 2u)m}{2},$$
$$C = \lambda^{-1}(u + u_x + 1) + \frac{\lambda(u^2 - u_x^2 + 2u)m}{2},$$
$$\begin{aligned} U &= \begin{pmatrix} \frac{-1}{2} & \frac{\lambda m}{2} \\ \frac{-\lambda m}{2} & \frac{1}{2} \end{pmatrix}, \\ V &= \begin{pmatrix} A & B \\ C & -A \end{pmatrix}. \end{aligned}$$

We construct a parametric representation of the smooth solution of problem (2)-(3) in terms of the solution of an associated Riemann–Hilbert problem, which can be efficiently used for further studying the properties of the solution. Particularly, using the proposed formalism, we describe regular as well as non-regular (cuspon and loop-shaped) one-soliton solutions [6] corresponding to the Riemann–Hilbert problems with trivial jump conditions and appropriately chosen residue conditions.

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Hereditarily Baire Hyperspaces

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A topological space X is Baire if the intersection of a countable family of open dense sets in X is dense. We say that X is hereditarily Baire if every closed subspace of X is Baire.

In this series of talks, I will focus on the following problem: Let X be a separable metric space. When the hyperspace $K(X)$ of all nonempty compact subsets of X endowed with the Vietoris topology is hereditarily Baire? A satisfactory answer to the above question was recently given by Gartside, Medini and Zdomskyy who observed its connection with a property of the remainder of some (any) compactification of X (the Menger property). I will show how topological games can help to prove quite easily this theorem. Next, I will give some applications of our techniques to spaces of probability measures and filters on the naturals.

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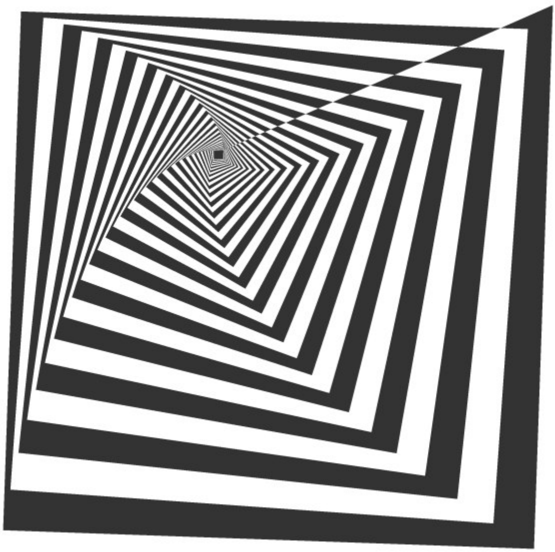

Cauchy completeness for metric spaces and enriched categories

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We introduce the notion of a category enriched in a monoidal category [1] via the example of metric spaces. Generalized metric spaces of Lawvere [2] are discussed in detail. For them Cauchy completeness (in the sense of Lawvere [2]) is equivalent to the property that every fundamental sequence converges to at least one point. For ordinary categories (enriched in Set) Cauchy completeness is equivalent to the property that every idempotent splits. Denote by Ab the monoidal category of abelian groups. An Ab-category is Cauchy complete iff it admits finite direct sums and every idempotent splits. Thus, the notion of completeness relates analysis and algebra.

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Introduction to homotopy theory

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The aim of these three lectures is to explain some basic homotopy invariants of topological spaces and illustrate their computaions. We will discuss the following topics:

1) Notion of homotopy. Homotopy equivalences and homotopy type. Retracts and deformational retracts.
2) Fundamental group and higher homotopy groups of a topological space.
3) Long exact sequence of homotopy groups of a pair of topological spaces.
4) Serre fibrations. Long exact sequence of homotopy groups for Serre fibrations.
5) Compact open topologies. Homotopies as paths in functional spaces.
6) Space of paths and loop space. Relations between homotopy groups of a space and its loop space.
7) Seifert - van Kampen theorem.
8) Gluing one space to another by a continuous map. CW-complexes. Construction of a compact topological space with given finitely presented fundamental group.

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Self-similar singular function defined by double stochastic matrices

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Let

1) $1=\frac{2}{3}+\sum\limits_{n=1}^{\infty}\frac{1}{(-2)^n}=\frac{2}{3}-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\ldots$ be normalized alternating binary series that defines a binary negapositional image of the number of the segment $[0;1]$:

$$x=\frac{2}{3}+\frac{\alpha_1(x)}{(-2)^1}+\frac{\alpha_2(x)}{(-2)^2}+\frac{\alpha_3(x)}{(-2)^3}+\ldots+\equiv\overline{\Delta}^2_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots};$$

2) $\|p_{ik}\|=\begin{pmatrix}p_{00} & p_{01}\\ p_{10} & p_{11}\end{pmatrix}$ be a positive double stochastic matrix i.e. $p_{ij}>0,\ p_{i0}+p_{i1}=1,\ p_{0j}+p_{1j}=1,\ i=0,1,\ j=0,1;$

3) $\overline{p}=(p_0;p_1)$ be a vector $p_0=\frac{p_{10}}{p_{01}+p_{10}}=\frac{1}{2}$ and $p_1=\frac{p_{01}}{p_{01}+p_{10}}=\frac{1}{2}$.

It is known that a binary negapositional number representation is a recoding of a classical binary representation:
$x=\frac{a_1}{2}+\frac{a_2}{2^2}+\ldots+\frac{a_n}{2^n}+\ldots\equiv\Delta^2_{a_1(x)a_2(x)\ldots a_n(x)\ldots},\quad a_n\in\{0;1\}$
Considered in the talk is function F, defined by equality

$$F(x)=F(\overline{\Delta}^2_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots})= \tag{1}$$
$$=\beta_{\alpha_1(x)}+\frac{1}{2}\sum_{k=1}^{\infty}(\beta^{(k)}_{\alpha_k(x)\alpha_{k+1}(x)}\prod_{i=1}^{k-1}p_{\alpha_i(x)\alpha_{i+1}(x)}),$$

$$\beta_{\alpha_1(x)}=\begin{cases}0, & \text{if}\quad \alpha_1(x)=1,\\ \frac{1}{2}, & \text{if}\quad \alpha_1(x)=0,\end{cases}$$

$$\beta^{(2n-1)}_{\alpha_{2n-1}(x)\alpha_{2n}(x)}=\beta^{(1)}_{\alpha_{2n-1}(x)\alpha_{2n}(x)}=\begin{cases}0, & \text{if}\quad \alpha_{2n}(x)=0,\\ p_{00}, & \text{if}\quad \alpha_{2n-1}(x)\neq\alpha_{2n}(x)=1,\\ p_{10}, & \text{if}\quad \alpha_{2n-1}(x)=\alpha_{2n}(x)=1,\end{cases}$$

$$\beta^{(2n)}_{\alpha_{2n}(x)\alpha_{2n+1}(x)} = \beta^{(0)}_{\alpha_{2n}(x)\alpha_{2n+1}(x)} = \begin{cases} 0, & \text{if} \quad \alpha_{2n+1}(x) = 1, \\ p_{01}, & \text{if} \quad \alpha_{2n}(x) = \alpha_{2n+1}(x) = 0, \\ p_{00}, & \text{if} \quad \alpha_{2n}(x) \neq \alpha_{2n+1}(x) = 0, \end{cases}$$

and $\alpha_k(x)$ is k negapositional digit of representation of the number x.

Definition 1. *Let $(c_1, c_2, \ldots, c_m)$ be a orderly set of positive integers.* The Cylinder of m rank with basis $c_1c_2\ldots c_m$ *is called a set* $\overline{\Delta}^2_{c_1c_2\ldots c_m}$ *of numbers of $x \in (0;1]$ that is first m negapositional digits of which are $c_1, c_2, \ldots, c_m$ respectively, i.e.*

$$\overline{\Delta}^2_{c_1c_2\ldots c_m\ldots} =$$

$$\left\{x: \; x = \overline{\Delta}^2_{c_1c_2\ldots c_m a_{m+1}a_{m+2}\ldots}, \quad a_{m+i} \in \mathbb{N}, \; i = 1, 2, 3, \ldots\right\}.$$

Lemma 1. *For a function F defined by the equality (1) the mapping of the cylinder $\overline{\Delta}^2_{c_1c_2\ldots c_m}$ is a segment $[a;b]$, where*

$$a = \beta_{c_1} + \frac{1}{2}\sum_{k=1}^{m-1}\left(\beta^{(k)}_{c_kc_{k+1}}\prod_{j=1}^{k-1} q_{c_jc_{j+1}}\right), \qquad b = a + \frac{1}{2}\prod_{j=1}^{m-1} q_{c_jc_{j+1}},$$

Theorem 2. *Images of different cylinders of the same rank with the mapping F do not overlap and in the union give the whole segment* $[0,1]$.

Theorem 3. *The function $F(x)$ denoted by the equality (1) is:*

1) *correctly identified,*

2) *continuous,*

3) *strictly increasing,*

4) *linear for $p_{00} = 0.5$ and singular for $p_{00} \neq 0.5$ (has a derivative equal to zero almost everywhere in the sense of the Lebesgue measure).*

Lemma 4. *The right-side shift operator $\delta_{ij}(x)$ of $\overline{\Delta}^2$-representation of a number with parameters $(i;j)$ denoted by an equality*

$$\delta_{ij}(x) = \delta_{ij}(\overline{\Delta}^2_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots}) = \overline{\Delta}^2_{ij\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots},$$

is given by the formula

$$\delta_{ij}(x) = \frac{1-i}{2} + \frac{j}{2^2} + \frac{1}{2^2}x.$$

Theorem 5. *The function $F(x)$ satisfies the system of functional equations:*

$$F(\delta_{ij}(x)) =$$

$$= \begin{cases}
F(\frac{1}{4}x + \frac{1}{3}) = \frac{1}{2}p_{01}^2 - \frac{1}{2}p_{01}p_{00} + p_{01}p_{00}F_0(x), & \textit{if } x = \overline{\Delta}^2_{0\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{100\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x + \frac{1}{2}) = \frac{1}{2} + \frac{1}{2}p_{00}p_{01} - \frac{1}{2}p_{00}^2 + p_{00}^2F_0(x), & \textit{if } x = \overline{\Delta}^2_{0\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{000\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x + \frac{3}{4}) = \frac{1}{2} + \frac{1}{2}p_{00} + \frac{1}{2}p_{00}p_{01} - \frac{1}{2}p_{01}^2 + p_{01}^2F_0(x), & \textit{if } x = \overline{\Delta}^2_{0\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{010\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x + \frac{1}{4}) = \frac{1}{2}p_{01} + \frac{1}{2}p_{00}^2 - \frac{1}{2}p_{00}p_{01} + p_{00}p_{01}F_0(x), & \textit{if } x = \overline{\Delta}^2_{0\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{110\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x + \frac{1}{2}) = \frac{1}{2} + p_{00}p_{01}F_1(x), & \textit{if } x = \overline{\Delta}^2_{1\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{001\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x + \frac{3}{4}) = \frac{1}{2} + \frac{1}{2}p_{00} + p_{00}p_{01}F_1(x), & \textit{if } x = \overline{\Delta}^2_{1\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{011\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x) = p_{01}^2F_1(x), & \textit{if } x = \overline{\Delta}^2_{1\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{101\alpha_2\alpha_3\ldots}, \\
F(\frac{1}{4}x + \frac{1}{4}) = \frac{1}{2}p_{01} + p_{00}^2F_1(x), & \textit{if } x = \overline{\Delta}^2_{1\alpha_2\alpha_3\ldots}, \\
 & \delta_{ij}(x) = \overline{\Delta}^2_{111\alpha_2\alpha_3\ldots}.
\end{cases}$$

The report proposes the results of investigation of the above-mentioned functions.

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On invariant closed convex sets of probability measures

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By $P(X)$ we denote the space of probability measures on a compact Hausdorff space X, by $\exp(X)$ the hyperspace of X.

By $ccP(X)$ we denote the hyperspace of closed convex subsets of X. It is known that ccP is a functor on the category **Comp** of compact Hausdorff spaces [5]. Moreover, ccP determines a monad in the category **Comp**; we denote by $\theta = (\theta_X)\colon (ccP)^2 \to ccP$ its multiplication.

Now let (X, d) be a compact metric space. The Kantorovich metric on the space of probability measures and the Hausdorff metric on the corresponding hyperspace determine the metric on the space ccP; we denote it by $\tilde{d}$. The map $\theta_X\colon (ccPccP(X), \tilde{\tilde{d}}) \to (ccP(X), \tilde{d})$ is known to be non-expanding [5].

Let $\{f_1, f_2, \dots, f_n\}$ be a finite family of contractions on X (that is, an iterated function system, IFS). Let also $B \in ccP(\{1, 2, \dots, n\})$. Define the map $\Phi_B\colon ccP(X) \to ccP(X)$ as follows. Let $A \in ccP(X)$ and let $g_A\colon \{1, 2, \dots, n\} \to ccP(X)$ be the map sending i to $ccP(f_i)(A)$; then let

$$\Phi_B(A) = \theta_X(ccP(g_A)(B)).$$

Theorem 1. *For any IFS $\{f_1, f_2, \dots, f_n\}$ and $B \in ccP(\{1, 2, \dots, n\})$ there exists a unique invariant closed convex set of probability measures, i.e., there exists $A \in ccP(X)$ such that $A = \Phi_B(A)$.*

A standard proof of this statement is based on the Banach Contraction Principle; another approach follows the line of the main result of [4].

There exists an inhomogeneous counterpart of invariant closed set of probability measures. Suppose that we have an IFS $\{f_1, f_2, \dots, f_n\}$ on X, $B \in ccP(\{0, 1, 2, \dots, n\})$ and $C \in ccP(X)$. For any $A \in ccP(X)$, let $g'_{A,C}\colon \{0, 1, 2, \dots, n\} \to ccP(X)$ be defined by the formula: $g'_{A,C}(0) = C$, $g'_{A,C}(i) = ccP(f_i)(A)$, $i = 1, 2, \dots, n$. Define

$\Phi'_{B,C}\colon ccP(X) \to ccP(X)$ by the formula

$$\Phi'_{B,C}(A) = \theta_X(ccP(g'_{A,C})(B)).$$

Theorem 2. *For any IFS* $\{f_1, f_2, \ldots, f_n\}$, $B \in ccP(\{0, 1, \ldots, n\})$ *and* $C \in ccP(X)$ *there exists a unique* $A \in ccP(X)$ *such that* $A = \Phi'_{B,C}(A)$.

The set A is called an *inhomogeneous invariant* convex set of probability measures.

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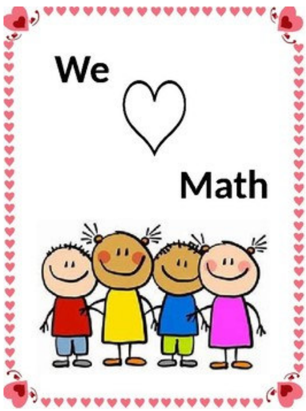

Compactness and completeness in partial metric spaces

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A function $q : X^2 \to [0, +\infty)$ is called *a quasi-pseudometric* on a set X if for every $x, y, z \in X$ the following conditions

(q_1) $q(x, x) = 0$;

(q_2) $q(x, z) \leq q(x, y) + q(y, z)$.

are true.

Let (X, q) be a quasi-pseudometric space. For every $x \in X$ the balls

$$B_q(x, \varepsilon) = \{y \in X : q(x, y) < \varepsilon\}, \quad \varepsilon > 0$$

form a base of quasi-pseudometric topology at the point x.

A quasi-pseudometric $q : X^2 \to [0, +\infty)$ is called *a quasi-metric* on X if $x = y \Leftrightarrow q(x, y) = q(y, x) = 0$ for every $x, y \in X$; and *an asymmetric metric* on X if $x = y \Leftrightarrow q(x, y) = 0$ for every $x, y \in X$.

A function $p : X^2 \to [0, +\infty)$ is called *a partial metric* on X if for every $x, y, z \in X$ the following conditions

(p_1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$;

(p_2) $p(x, x) \leq p(x, y)$;

(p_3) $p(x, y) = p(y, x)$;

(p_4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

are true.

For any partial metric $p : X^2 \to [0, +\infty)$ the function $q_p : X^2 \to [0, +\infty)$, $q_p(x, y) = p(x, y) - p(x, x)$, is a quasi-metric on X and the topology of the partial metric space (X, p) is the topology of the quasi-metric space (X, q_p). Moreover, the function $d_p : X^2 \to [0, +\infty)$, $d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y)$ is a metric on X.

A quasi-pseudometric space (X, q) is called *precompact* if for every $\varepsilon > 0$ there exists a finite set $A \subseteq X$ such that $X \subseteq \bigcup_{a \in A} B_q(a, \varepsilon)$.

A sequence $(x_n)_{n=1}^{\infty}$ in a quasi-pseudometric space (X, q) is said to be *left q-Cauchy* if for each $\varepsilon > 0$ there is a point $x \in X$ and an integer k such that $q(x, x_m) < \varepsilon$ for all $m \geq k$; and *left K-Cauchy* if for each $\varepsilon > 0$ there is an integer k such that $q(x_n, x_m) < \varepsilon$ for all $m \geq n \geq k$. A pseudo-quasimetric space (X, q) is said to be *left q-complete* if every left q-Cauchy sequence in X is convergent in X; and *left K-complete* if every left K-Cauchy sequence in X is convergent in X.

A sequence $(x_n)_{n=1}^{\infty}$ in a partial metric space (X, p) is called *Cauchy* if $\lim\limits_{m,n \to \infty} p(x_n, x_m)$ exists and is finite. A partial metric space (X, p) is called *complete* if every Cauchy sequence $(x_n)_{n=1}^{\infty}$ in X converges to a point $x_0 \in X$ such that $\lim\limits_{m,n \to \infty} p(x_n, x_m) = p(x_0, x_0)$.

Partial metrics p and r on a set X are called *equivalent* if the topologies of the spaces (X, p) and (X, r) coincide.

Theorem 1. *For any partial metric space (X, p) the following conditions are equivalent:*

(i) X is a countable compact space;

(ii) X is a sequentially compact space;

(iii) X is a compact space;

(iv) X is precompact and left q_p-complete;

(v) X is precompact and left K-complete.

Example 2. *There exists a Hausdorff compact partial metric space (X, p) which is not complete.*

Example 3. *There exists a compact partial metric space (X, p) such that the space (X, r) is not complete for any partial metric r which is equivalent to p.*

Example 4. *There exists a partial metric space (X, p) such that*

(1) (X, p) is completely regular, separable, perfect pseudocompact space;

(2) (X, p) *is not compact, consequently it is not normal and paracompact.*

Example 5. *Let* $X = \{x_n : n \in \mathbb{N}\}$ *and* $p(x_n, x_m) = \max\{n, m\}$ *for every* $n, m \in \mathbb{N}$*. Then* p *is not equivalent to any bounded complete partial metric on* X*.*

Theorem 6. *Let* (X, p) *be a complete partial metric space and* D *is the set of discontinuity points set of the mapping* $f : (X, p) \to \mathbb{R}$, $f(x) = p(x, x)$*. If the set* $f(D) = \{p(x, x) : x \in D\}$ *is bounded, then there exists a bounded complete partial metric* r *on* X *which is equivalent to* p*.*

Question 1. *Let* (X, p) *be a compact partial metric* T_1*-space. Is there an equivalent partial metric* r *on* X *such that* (X, r) *is complete?*

Question 2. *Is a complete partial* T_1*-metric* p *on* X *equivalent to a bounded complete partial metric on* X*?*

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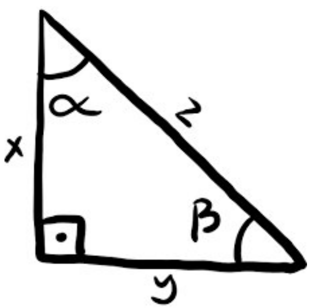

On the conditions for some entire functions to have only real zeros

Thu Hien Nguyen and **Anna Vishnyakova**
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Definition 1. *A real entire function f is said to be in the Laguerre-Pólya class, written $f \in \mathcal{L} - \mathcal{P}$, if it can be expressed in the form*

$$f(x) = cx^n e^{-\alpha x^2 + \beta x} \prod_{k=1}^{\infty} \left(1 - \frac{x}{x_k}\right) e^{x x_k^{-1}}, \qquad (1)$$

where $c, \alpha, \beta, x_k \in \mathbb{R}$, $x_k \neq 0$, $\alpha \geq 0$, n is a nonnegative integer and $\sum_{k=1}^{\infty} x_k^{-2} < \infty$. The product on the right-hand side can be finite or empty.

The Laguerre-Pólya class was studied by many authors, and its various significant properties and characterizations are mentioned in [1], [4] and other works. Note that for a real entire function (not identically zero) of order less than 2 having only real zeros is equivalent to belonging to the Laguerre-Pólya class.

The *partial theta-function*, $g_a(z) = \sum_{j=0}^{\infty} \frac{z^j}{a^{j^2}}$, $a > 1$, was studied in [2] and [3]. It is proved in [2] that there exists a constant $q_\infty, q_\infty \approx 3.23363666...$, such that the partial theta-function (and all its odd Taylor sections) belongs to the Laguerre-Pólya class if and only if $a^2 \geq q_\infty$.

We study the class of the entire functions with positive coefficients having monotonic second quotients of Taylor coefficients: $q_n = q_n(f) := \frac{a_{n-1}^2}{a_{n-2}a_n}$. We have obtained the sufficient and necessary condition for these functions to belong to the Laguerre-Pólya class.

Theorem 1. *(T. H. Nguyen, A. Vishnyakova, [5]). Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$ for all k, be an entire function. Suppose that $q_n(f)$ are decreasing in n, i.e. $q_2 \geq q_3 \geq q_4 \geq \ldots$, and $\lim_{n\to\infty} q_n(f) = b \geq q_\infty$. Then all the zeros of f are real and negative, in other words $f \in \mathcal{L} - \mathcal{P}$.*

Theorem 2. *(T. H. Nguyen, A. Vishnyakova). Let* $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$, *be an entire function. Suppose that the quotients* $q_n(f)$ *are increasing in* n, *and* $\lim_{n\to\infty} q_n(f) = c < q_\infty$. *Then the function* f *does not belong to the Laguerre-Pólya class.*

We also considered a special function

$$f_a(z) = \sum_{k=0}^{\infty} \frac{z^k}{(a+1)(a^2+1)\dots(a^k+1)}.$$

We discuss the necessary and the sufficient condition for it to belong to the Laguerre-Pólya class.

Theorem 3. *(T. H. Nguyen). The entire function*

$$F_a(z) = \sum_{k=0}^{\infty} \frac{z^k}{(a^k+1)(a^{k-1}+1)\cdot\ldots\cdot(a+1)}, a > 1,$$

belongs to the Laguerre-Pólya class if and only if there exists $z_0 \in (-(a^2+1); -(a+1))$ *such that* $F_a(z_0) \le 0$.

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A NOTE ON COMPACT-LIKE SEMITOPOLOGICAL GROUPS

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We give a talk on a few results related to separation axioms and automatic continuity of operations in compact-like semitopological groups. In particular, is presented a semiregular semitopological group G which is not T_3. We show that each weakly semiregular compact semitopological group is a topological group. On the other hand, constructed examples of quasiregular T_1 compact and T_2 sequentially compact quasitopological groups, which are not paratopological groups. Also we prove that a semitopological group (G, τ) is a topological group provided there exists a Hausdorff topology $\sigma \supset \tau$ on G such that (G, σ) is a precompact topological group and (G, τ) is weakly semiregular or (G, σ) is a feebly compact paratopological group and (G, τ) is T_3.

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Categorical properties of functionals generated by the triangular norms

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We consider the class of functionals introduced in [1]. Note that some applications of these functionals can be found in [2].

A triangular norm (t-norm) is a continuous, associative, commutative and monotonic operation on the unit segment for which 1 is a unit.

The following are examples of t-norms:

1. $a * b = ab$;
2. $a * b = \min\{a, b\}$;
3. $a * b = \max\{a + b - 1, 0\}$ (Łojasiewicz t-norm).

Let X be a compact Hausdorff space. A functional $\mu\colon C(X, [0,1]) \to [0,1]$ is a $*$-measure if

1. $\mu(c_X) = c$;
2. $\mu(\lambda * \varphi) = \lambda * \mu(\varphi)$;
3. $\mu(\varphi \vee \psi) = \mu(\varphi) \vee \mu(\psi)$,

for all $c \in [0,1]$ and $\varphi, \psi \in C(X, [0,1])$. (Here, $\vee$ denotes the maximum.)

We show that every $*$-measure is a continuous map. The space $M^*(X)$ of the $*$-measures on X is compact Hausdorff. Actually, M^* is a functor in the category **Comp** of compact Hausdorff spaces.

Let exp denote the hyperspace functor in **Comp**. Denote by $\bar{M}(X)$ the set of all $A \in \exp(X \times \mathbb{I})$ satisfying the following conditions:

1. $A \cap (X \times \{1\}) \neq \emptyset$;
2. $X \times \{0\} \subset A$;

3. A is saturated, i.e., if $(x,t) \in A$, then $(x,s) \in A$ for every $s \in [0,t]$.

Let $f\colon X \to Y$ be a map. Define the map $\bar{M}(f)\colon \bar{M}(X) \to \bar{M}(Y)$ by the formula:

$$\bar{M}(f)(A) = \exp(f \times 1_{\mathbb{I}})(A) \cup (Y \times \{0\}).$$

Actually, $\bar{M}$ is a functor in the category **Comp**. We establish relations between the functors M^* and $\bar{M}$. Also, we consider the monads generated by these functors.

Our results are extensions of those obtained in [3].

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Application of absolutely summing operators to isomorphic classification of Banach spaces of differentiable functions

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We give a brief introduction to the theory of absolutely summing operators as operator ideals on Banach spaces. We present several proofs of remarkable theorem of Grothendieck on operators from L^1 spaces to the Hilbert space. Then we show how Grothendieck's theorem could be applied to prove that the Sobolev space of functions with integrable gradient is not isomorphic to L^1 space.

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Non-expansive bijections between unit balls of Banach spaces and related problems

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A metric space M is called expand-contract plastic if every bijective non-expansive (BnE for short) map $F: M \to M$ is an isometry. There is a number of publications devoted to studying of this property [2], [3], [4]. In particular, Theorem 2.6 from [2] states that the unit ball of every strictly convex Banach space is plastic. In [5] more general question was considered.

Question 1. *Let X, Y be Banach spaces and let $F: B_X \to B_Y$ be a BnE map. For which Banach spaces F turns out to be an isometry?*

In the same paper, continuing the result from [2], this question was answered in positive for strictly convex space Y and arbitrary X. We get the analogous theorem, where the roles of spaces X and Y are inverted.

Theorem 1. *Let X, Y be Banach spaces, X be strictly convex and $F : B_X \to B_Y$ be a BnE map. Then F is an isometry.*

The result of Theorem 2 is based on the facts that the unit sphere of any strictly convex space consists of extreme points, and the preimage of any extreme point under BnE map is also an extreme point. We were able to get the following generalization.

Theorem 2. *Let X, Y be Banach spaces, $F : B_X \to B_Y$ be a BnE map, then for every $n \in \mathbb{N}$ the preimage of any n-dimensional convex polyhedral extreme subset $C \subset S_Y$ is an n-dimensional convex polyhedral extreme subset of S_X. Moreover, the equality $-F^{-1}(-C) = F^{-1}(C)$ holds true.*

Theorem 3. *Let X, Y be Banach spaces, $F : B_X \to B_Y$ be a BnE map and S_Y be the union of all its finite-dimensional polyhedral extreme subsets. Then F is an isometry.*

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ON FIRST COUNTABLE T_1 LINDELÖF SPACES

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The talk will be devoted to problems motivated by the following celebrated result of Arkhangelskii: Every first countable compact Hausdorff space has cardinality at most $2^{\aleph_0}$.

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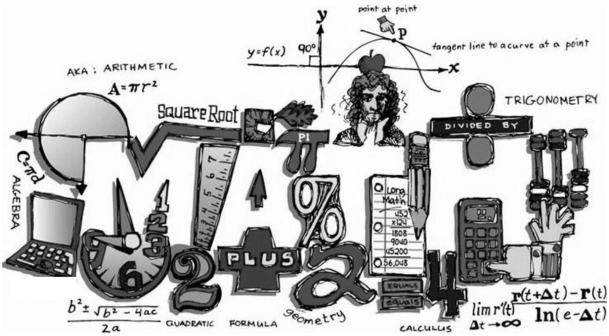

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